Charge to Mass Ratio of an Electron

Objective：

According to moving charged particles could be deflected in a magnetic field, we can measure the charge to mass ratio of an electron.

Apparatus：

$\frac{e}{m}$ experiment apparatus (include the electron beam tube and Helmholtz Coils)

Principle：

In 1897, J.J. Thompson set out to prove that the cathode produced a stream of negatively charged particles called electrons. The beam of electrons in the tube is produced by an electron gun composed of a heated filament. Moving charged particles could be deflected in a magnetic field. It provides a method of measuring e/m, charge to mass ratio of an electron.

In 1909, Robert Millikan determined the elementary charge *e*. Then using Thompson's value of e/m, he calculated the value of *m*.

Assume that the electron is initially at rest and is accelerated through an electric potential difference *V*. The electron has a kinetic energy: (Assume charge *e* moving with velocity v)

$$\text{K.E.} = eV = \frac{1}{2}mv^2 \tag{1}$$

As shown in Figure 1. The magnetic force acting on a charged particle of charge *e* moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by

$$\vec{F} = -e\vec{v} \times \vec{B} \tag{2}$$

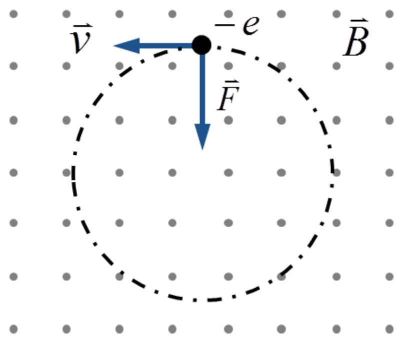


Figure 1. Electron moved in the magnetic field

The magnetic field vector is perpendicular to the paper surface, directed out of the paper; and the charged particle velocity is perpendicular to $\vec{B}$. According to the "right-hand rule", the magnitude force $\vec{F}$ is perpendicular to $\vec{v}$. It implies that the particle trajectory is circular, and the particle must be experiencing uniform circular motion by a centripetal force of magnitude.

Now

$$\left|\vec{F}\right| = ma = \frac{mv^2}{r} \tag{3}$$

Where *m* is the particle mass, *a* is acceleration of electron and *r* is the radius of the circular motion. Assume that the moving particle is an electron with the charge *e*.

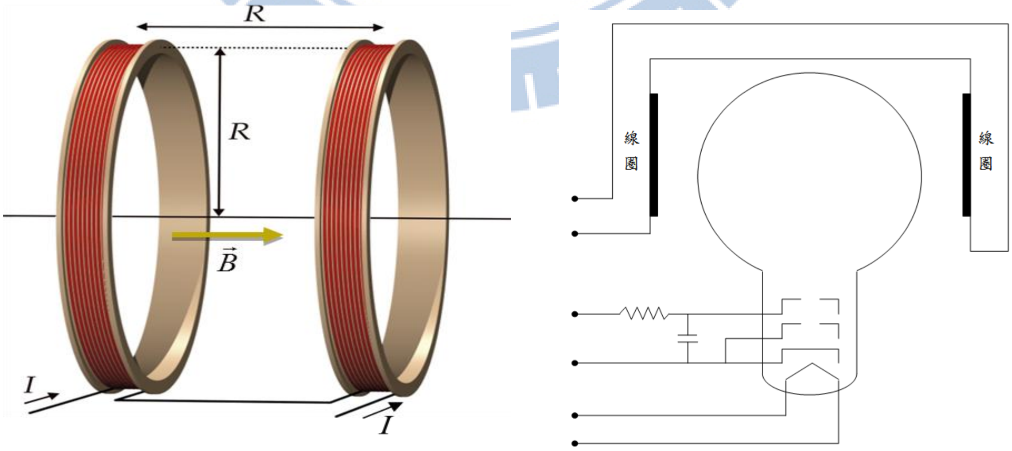
Combine equation (2) and (3), we can get the equation

$$m\frac{v^2}{r} = evB \tag{4}$$

Substitute equation (1) into equation (4), we can get the charge to mass ratio of the electron

$$\frac{e}{m} = \frac{2V}{B^2 r^2} \tag{5}$$

In this experiment, the uniform magnetic field is generated by Helmholtz Coils. As shown in figure 2(a). Helmholtz Coils are a pair of flat circular coils, with equal numbers of turns and equal diameters, arranged with a common axis and connected in series.



(*a*) (*b*)

Figure 2. Helmholtz Coils and circuit diagram

After Helmholtz Coils set up an electric current, they generate a nearly uniform magnetic field, parallel to the center line of two coils. The magnetic field is given by Biot–Savart Law and express as

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \mu_0 \frac{NI}{R} = \frac{8\mu_0 NI}{\sqrt{125}R} \tag{6}$$

that

μ_0	permeability Constant $\mu_0 = 4\pi \times 10^{-7}$ (Weber/Amp · m)
N	coil turns ($N = 145$)
I	the current of coil
R	coil radius ($R = 14$ cm)

Where N is the number of turns of one coil, R is the mean radius of the coil; i is the current through the coils and μ_0 is the permeability of free space. Combining equation (5) with equation (6) yields

$$\frac{e}{m} = \frac{125R^2V}{32{\mu_0}^2 N^2 I^2 r^2} \tag{7}$$

[Note] The theoretical value of charge to mass ratio of an electron $\frac{e}{m} = 1.76 \times 10^{11}$ (C/kg)

<u>Remarks</u>：

1. If the operation lasts over one hour, take a rest for a while.
2. Before turning on the power, make sure that the voltage and current controls are both turned down to zero.
3. Slowly vary the plate voltage to prevent damage to the apparatus.

<u>Procedure</u>：

- **Preparation**
  1. Turn of the power, Set the deflector voltage (DEFLECTING VOLTAGE) to OFF.
  2. Set the current direction in the magnetizing coil to CLOCKWISE.
  3. Slowly turn the plate voltage control clockwise to increase the voltage until an electron beam appears.
  4. Turn on the electron beam tube let incident electron beam is perpendicular to the magnetic field.

A. The incident electron beam is perpendicular to the magnetic field

(a) Keep the plate voltage constant

1. Keep the plate voltage constant. (i.e. Fix it at 120-200 V) And find the radius *r* of the electron beam with different current(less than 2.0A) through the Helmholtz Coils.
2. Record the radius *r* corresponding to the different currents.
3. Calculate the charge to mass ratio of an electron $\frac{e}{m}$.
4. Plot $\frac{1}{r^2} - I^2$ diagram.

(b) Keep the current through the Helmholtz Coils constant

1. Keep the current through the Helmholtz Coils constant. (i.e. Fix it at 2.0 A) And find the radius *r* of the electron beam with different the plate voltages.
2. Record the radius *r* corresponding to the different voltage.
3. Calculate the charge to mass ratio of an electron $\frac{e}{m}$.
4. Plot $\frac{1}{r^2} - \frac{1}{V}$ diagram.

B. The incident electron beam is not perpendicular to the magnetic field.

1. Set the voltage between 150 V and the current between 2.0 A.
2. Keep the voltage and current constant. Find the radius *r* of the electron beam with different incident angle θ of the electron beam.
3. Record the radius *r* corresponding to the different incident angle.
4. When the electron beam is not perpendicular to the magnetic field, the charge to mass ratio of an electron is $\frac{e}{m} = \frac{125R^2V\left(\sin^2\theta\right)}{32\mu_0^2N^2I^2r^2}$.
5. Calculate the charge to mass ratio of an electron $\frac{e}{m}$.

Questions：

1. Why can we use Helmholtz Coils to generate a magnetic field in this experiment? Please explain.
2. Will the Earth's magnetic field affect the electron beam trajectory? Please explain.