Unit-05 Collision Experiment

Objective：

Place two trolleys on the nearly frictionless aluminum trajectory and carry out elastic collisions and inelastic collisions between the two trolleys to verify the law of momentum conservation

Apparatus：

Aluminum trajectory, trolley, webcam, weight, level, isotropic magnet, upside-down U-shaped spring and buffer board, velcro, tip and clay, electronic scales

Principle：

When two particles collide in a closed (no mass enters or leaves) and isolated (no net external force acting on particles) system. The total momentum of the two particles remains the same before and after collisions, and thus the momentum is conserved. The total kinetic energy is conserved only in elastic collisions. The following will briefly explain the changes of momentum and kinetic energy of the two trolleys before and after one dimension elastic collisions, inelastic collisions and perfectly inelastic collisions on a nearly frictionless trajectory.

In this experiment, m_1 is the trolley's mass. m_2 is the total mass of the weights on the trolley and the anti-dazzling board. Set the initial velocities of two trolleys as $v_1$and v_2, and the velocities of two trolleys after collision as u_1 and u_2.

A. Elastic Collisions

Fig. 1 shows an elastic collision between two trolleys on a nearly frictionless trajectory. Under this condition, the two trolleys must obey the laws of momentum conservation and kinetic energy conservation before and after collision. The equations can be represented as

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

By solving these simultaneous equations, we would get

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$u_2 = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

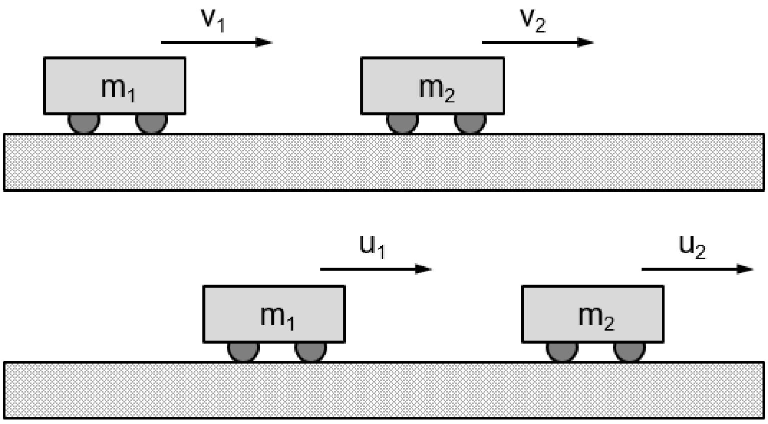

Figure 1. Schematic representation of an elastic collision between two trolleys

If a trolley in motion m_1 collides with a still ($v_2 = 0$) trolley m_2, then

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$u_2 = \frac{2m_1}{m_1 + m_2} v_1$$

Here, we define all the velocities of the trolley: right hand side as positive, left hand side as negative. If $v_1 > 0$ and $v_2 = 0$, then we can discuss the elastic collisions in three special conditions.

[Note] Right hand side as a positive value；Left hand side as a negative value

1. When $m_1 = m_2$, and after collision $u_1 = 0$ and $u_2 > 0$. Two trolleys exchange their motions and velocities after collision.
2. When $m_1 > m_2$, and after collision $u_1 > 0$ and $u_2 > 0$. Two trolleys move in the same direction after collision.
3. When $m_1 < m_2$, and after collision $u_1 < 0$ and $u_2 > 0$. Two trolleys move in opposite directions after collision.

B. Inelastic Collisions

Fig. 2 shows a perfectly inelastic collision between two trolleys on a nearly frictionless trajectory. Perfectly inelastic collision happened when after collisions, two trolleys stick together and move in the same velocity $(u_1 = u_2 = u)$ toward the same direction. Under this condition, only momentum was conserved between two trolleys before and after collision. The equations can be represented as

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\Rightarrow m_1 v_1 + m_2 v_2 = (m_1 + m_2) u$$

If a trolley in motion m_1 collides a still ($v_2 = 0$) trolley m_2, then

$$u = u_1 = u_2 = \frac{m_1}{m_1 + m_2} v_1$$

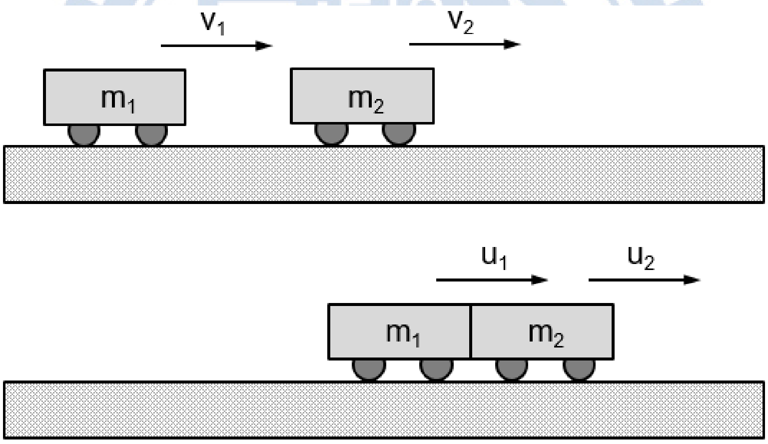

Figure 2. Schematic representation of a perfectly inelastic collision between two trolleys

C. Recover Coefficient

If the total kinetic energy of two trolleys is not conserved before and after collisions, this is called an inelastic collision. We can define

$$e = -\frac{u_2 - u_1}{v_2 - v_1}$$

1. When $e = 0$, it is a perfectly inelastic collision
2. When $0 < e < 1$, it is an inelastic collision
3. When $e = 1$, it is an elastic collision.

(a) The rate of momentum loss

In elastic collisions and inelastic collisions, the total momentum of two trolleys should be conserved. During the experiment, the trolleys may be affected by external force. That we can calculate the rate of momentum loss according to the total momentum of the two trolleys before and after collision as following

$$\Delta P(\%)=\frac{(m_1v_1+m_2v_2)-(m_1u_1+m_2u_2)}{m_1v_1+m_2v_2}\times 100\%$$

(b) The rate of kinetic energy loss

In inelastic collisions and perfectly inelastic collisions, the total kinetic energy of two trolleys would not be conserved. Now, we can calculate the rate of kinetic energy loss according to the total kinetic energies of the two trolleys before and after collision as following

$$\Delta E(\%)=\frac{(m_1v_1^2+m_2v_2^2)-(m_1u_1^2+m_2u_2^2)}{m_1v_1^2+m_2v_2^2}\times 100\%$$

<u>Remarks</u>：

1. Level the trajectory, and tighten the screws before starting an experiment.
2. Connect the webcam to your personal computer and place the webcam in front of the trajectory so that the webcam can film the motion of a trolley on the trajectory.
3. Take all the velocities of a trolley: **right hand side as a positive value, left hand side as a negative value**.

<u>Procedure</u>：

➢ **Preparation**

1. Place the level on the trajectory. Adjust the screws on both sides underneath the trajectory to level it.
2. Place the trolley on the trajectory. Observe whether the trolley slides toward one side or the other of the trajectory. If so, please level the trajectory again until the trolley stops sliding. If it doesn’t slide, proceed with the next correction.
3. Place the webcam in front of the trajectory (shown in Fig. 3). Use vision acquisition software to make sure you can acquire clear images of the two trolleys before and after collisions.
4. Set the parameters by instruction book.

A. Elastic collisions

Fig. 3 shows the vertical view of the set-up of trolleys' elastic collisions. $m_1$is the trolley's mass. m_2 is the total mass of the weights on the trolley and the anti-dazzling board. Webcam is the internet camera. One can use an upside-down U-shaped spring and a buffer film to form an isotropic magnet that serves as a bumper.

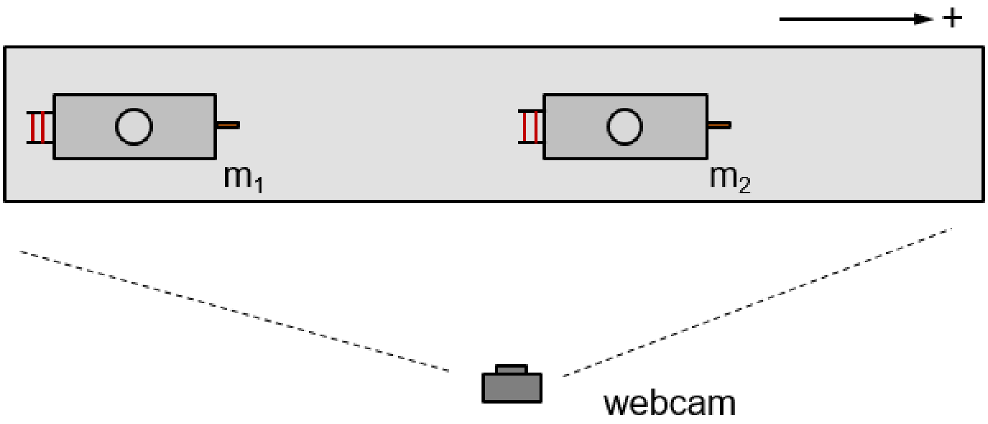

Figure 3. The vertical view of the set-up of trolley's elastic collisions

(a) Two trolleys with same mass

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the anti-dazzling board) with an electronic scale. Keep the total mass of the two trolleys the same, $m_1 = m_2$.
2. Place the trolley m_2 on the trajectory. (as shown in Fig. 3)
3. Use appropriate force to push trolley m_1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps three times.
6. Calculate the total momentum and total kinetic energy before and after collision, and coefficient of restitution as well as the efficiency of losing kinetic energy from data.

(b) A heavy trolley collides with a light trolley

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the anti-dazzling board) with an electronic scale. Keep the total mass of the two trolleys satisfying $m_1 > m_2$.
2. Place the trolley m_2 on the trajectory. (as shown in Fig. 3)
3. Use appropriate force to push trolley m_1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps three times.

6. Calculate the total momentum and total kinetic energy before and after collision, and coefficient of restitution as well as the efficiency of losing kinetic energy from data.

(c) A light trolley collides with heavy trolley

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the anti-dazzling board) with an electronic scale. Keep the total mass of the two trolleys satisfying $m_1 < m_2$.
2. Place the trolley m_2 on the trajectory.（as shown in Fig. 3)
3. Use appropriate force to push trolley m_1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps three times.
6. Calculate the total momentum and total kinetic energy before and after collision, and coefficient of restitution as well as the efficiency of losing kinetic energy from data.

B. Inelastic collisions

Fig. 4 shows the vertical view of the device of trolley's inelastic collisions. m_1 is the trolley's mass. m_2 is the total mass of the weights on the trolley and the anti-dazzling board. Webcam is the internet camera. One can use an upside-down U-shaped spring and a buffer film to form an isotropic magnet that serves as a bumper. One can use a Velcro or an anisotropic magnet that served as a bumper.

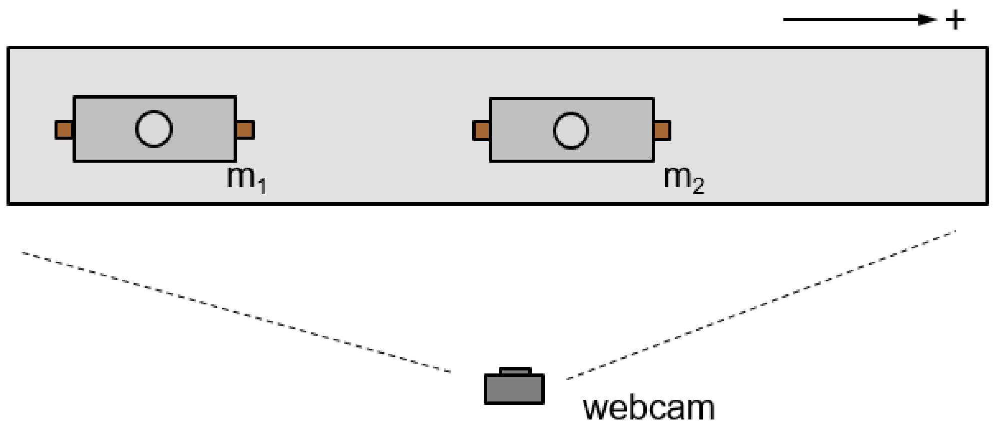

Figure 4. The vertical view of the set-up of trolley's inelastic collisions

(a) Two trolleys with same mass

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the anti-dazzling board) with an electronic scale. Keep the total mass of the two trolleys the same, $m_1 = m_2$.
2. Place the trolley m_2 on the trajectory.（as shown in Fig. 4)

3. Use appropriate force to push trolley m_1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps three times.
6. Calculate the total momentum and total kinetic energy before and after collision, and coefficient of restitution as well as the efficiency of losing kinetic energy from data.

(b) A heavy trolley collides with a light trolley

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the anti-dazzling board) with an electronic scale. Keep the total mass of the two trolleys satisfying $m_1 > m_2$.
2. Place the trolley m_2 on the trajectory. (as shown in Fig. 4)
3. Use appropriate force to push trolley m_1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps three times.
6. Calculate the total momentum and total kinetic energy before and after collision, and coefficient of restitution as well as the efficiency of losing kinetic energy from data.

(c) A light trolley collides with a heavy trolley

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the anti-dazzling board) with an electronic scale. Keep the total mass of the two trolleys satisfying $m_1 < m_2$.
2. Place the trolley m_2 on the trajectory. (as shown in Fig. 4)
3. Use appropriate force to push trolley m_1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps three times.
6. Calculate the total momentum and total kinetic energy before and after collision, and coefficient of restitution as well as the efficiency of losing kinetic energy from data.

Questions：

1. Prove the velocity-mass equation of the two trolleys after collisions with laws of momentum conservation and kinetic energy conservation.
2. If the trajectory is not leveled during a collision, would the system’s total momentum and kinetic energy conserve?
3. Please compare the recover coefficient between isotropic magnet, upside-down U-shaped spring and buffer board, velcro, tip and clay.
4. Following the previous question. Thy to compare common and difference between them.