Unit-08　Compound Pendulum Experiment

Objective：

Study the physical pendulum and calculate the free-fall acceleration constant by its properties.

Apparatus：

Physical pendulum, physical pendulum stent, photogate, digital device, photogate stent, level

Principle：

A physical pendulum is a pendulum that has a mass distribution. On the contrary, a simple physical pendulum has its mass in a plumb as a point. As shown in Figure 1, a physical pendulum is a rigid body of mass ***M*** hanged on a pivot **P**. The distance from **P** to the center of mass **C** is *h*.

When this object swings at a small angel, the torque from the gravity is

$$\tau = -Mgh \sin\theta$$

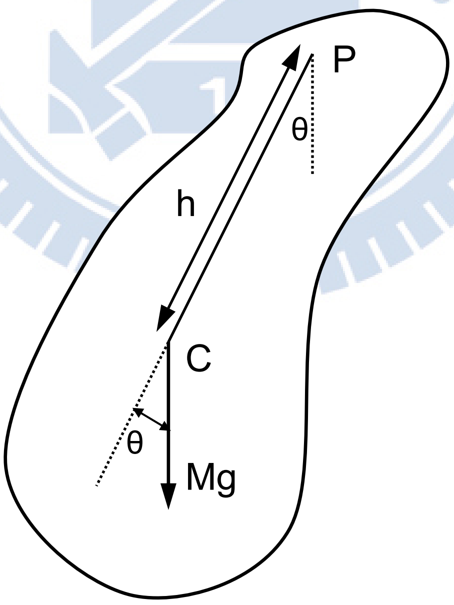


Figure 1. Pendulum illustration

Furthermore, if the angel is very small, the above equation can be simplified as

$$\tau = -(Mgh)\sin\theta \approx -Mgh\theta$$

The physical pendulum movement equation is

$$\tau = I\alpha = I\left(\frac{d^2\theta}{dt^2}\right)$$
$$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{\tau}{I} = -\frac{Mgh\theta}{I}$$
$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{Mgh}{I}\theta = 0$$

I is the moment of inertia and a is acceleration.

P is the pivot of the pendulum, that the swing situation is a simple harmonic motion (S.H.M).

$$\omega^2 = \frac{Mgh}{I}$$

ω is angular velocity

And its period of motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{Mgh}} \tag{1}$$

According to the parallel-axis theorem, the moment of inertia $I = I_{cm} + Mh^2$, we get

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{I_{cm} + Mh^2}{Mgh}} \tag{2}$$

I_{cm} is the moment of inertia corresponding to center-of-mass.

In this experiment, the sketch of the device is shown in Figure 2. A physical pendulum is composed of a metal rod and an adjustable plumb **D**. Suppose that the mass of this system is ***M***. **A** and **B** as the different pivots of the pendulum, the length of distance between two pivots is ***L***. If fix the plumb, the center of mass is at **C**. The distance between **A** to **C** is h_1, and the distance between **B** to **C** is h_2.

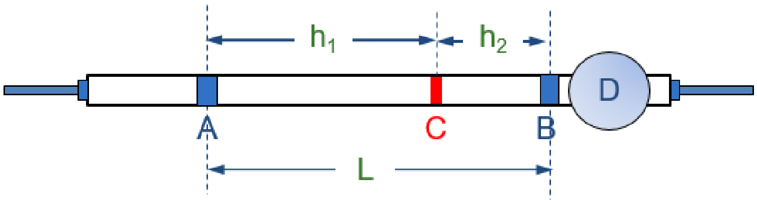
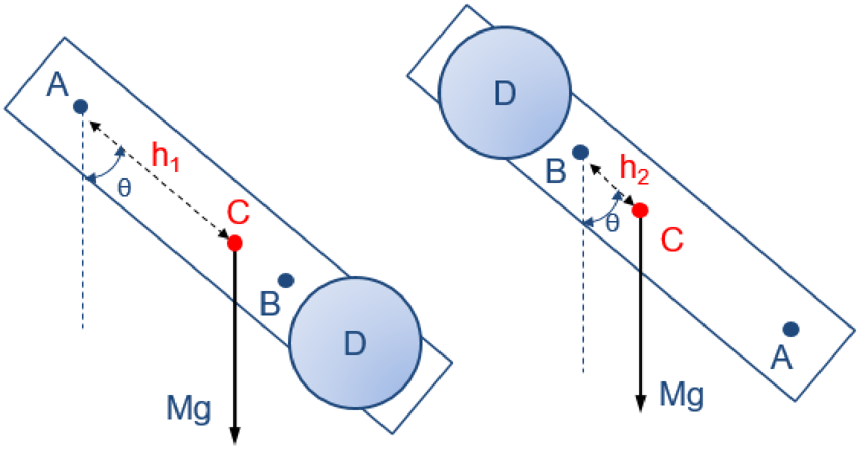


Figure 2. Pendulum structure

When fix the plumb at any position. Use **A** and **B** are the different pivots of the pendulum, our goal is to know the free-fall acceleration constant *g* by making the physical pendulum oscillates through two different pivots, even though we don't know the position of the center of mass. As shown in figure 3, we get the period T_A when the system swings by using **A** as the pivot. On the contrary, we get the period T_B by using **B** as the pivot.



(a) (b)

Figure 3. Pendulum for different pivots

Therefore, we have (From Eq. (2))

$$T_A = 2\pi\sqrt{\frac{I_A}{Mgh_1}} = 2\pi\sqrt{\frac{I_{cm} + Mh_1^2}{Mgh_1}} \tag{3}$$

$$T_B = 2\pi\sqrt{\frac{I_B}{Mgh_2}} = 2\pi\sqrt{\frac{I_{cm} + Mh_2^2}{Mgh_2}} \tag{4}$$

I_A and I_B are the moment of inertias corresponding to **A** and **B** respectively

We know the relations with I_A 、 I_B and I_{cm} by parallel-axis theorem. From Eq. (3) and Eq. (4) could get

$$g = \frac{4\pi^2\left(h_1^2 - h_2^2\right)}{\left(h_1 T_A^2 - h_2 T_B^2\right)} \tag{5}$$

Therefore, we can try to find the position where $T_A = T_B = T$ by adjusting the location of plumb and by regression. Finally, use Eq. (5) to calculate the free-fall acceleration constant g.

$$g = \frac{4\pi^2\left(h_1 + h_2\right)}{T^2} = \frac{4\pi^2 L}{T^2}$$

In there, $h_1 + h_2 = L$ is the distance of two different pivots.

Remarks：

1. Place your physical pendulum firmly on the stent.
2. If it shakes, please halt your experiment and recheck the set-up or adjust the screw on the rack to get better performance.
3. The physical pendulum is long and heavy, be careful when you move it.
4. Don't be naive swinging it as a lance. Beware of anything when experimenting.

Procedure：

➢ **Preparation**

1. Adjust the position of the photogate, so that the swing will swing through photogate.
2. Set the parameters by instruction book.

A. Measure the free-fall acceleration

1. Measure the length of distance between two pivots L.
2. Fix the plumb D at position S_1. (As the instruments ruler numerical)
3. Use **A** and **B** as the pivot, swing for 60 seconds respectively, and measure the period T_A and T_B.
4. Repeat above steps and shift the plumb **D** to S_1, S_2 ... S_5.
5. Plot $T-S$ diagram. (as shown in figure 4)

6. Make linear regression to both T_A and T_B. Solution of linear regression equations to find the point of intersection S'.
7. Adjust the plumb to S', and then repeat above steps. Check if you get the same periods on both swing by **A** and **B** as the pivot.
8. If not, slightly adjust the plumb until the result close enough, it should less than 0.004 seconds under a period.
9. Calculate free-fall acceleration *g*.

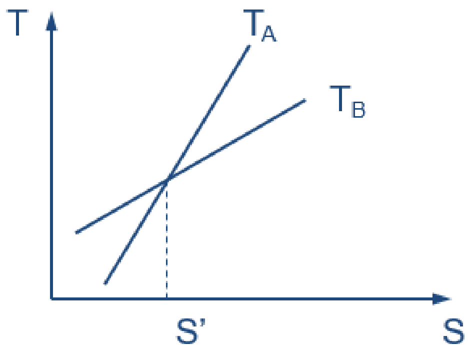


Figure 4. $T-S$ diagram.

Questions：

1. It is obvious that $T_A = T_B = T$ if $h_1 = h_2$. The question is that, can you make it in this experiment? Please explain.
2. If the pendulum D movable range increases, according to the experimental procedure resulting graph is linear? If not, this diagram should be what kind of relationship? Please explain.