Unit-05　RLC Series Circuit Experiment I

Objective：

In this experiment, we construct RLC series circuit to study the damped oscillation and the characteristic frequency.

Apparatus：

Oscilloscope, function generator, resistor, capacitor, inductor

Principle：

A system can oscillate when it has two ways of storing energy and the energy can flow alternately from one mode of storage to the other. In an electric circuit which contains an inductor L and a capacitor C, energy can be stored in the magnetic field of L or in the electrostatic field C. As the flow of current discharges C, a magnetic field is developed around L. As the magnetic field decays, the induced voltage causes C to charge with a polarity opposite to the original one. Then the process reverses. This switching frequency is called the eigenfrequency of the LC circuit. RLC circuit contains not only L and C but also a resistor R, which dissipates a certain fraction of the energy during each cycle. The eigenfrequency of RLC circuit is also the voltage versus time on the capacitor or the magnetic field versus time on the inductor.

In the RLC circuit we must provide a way of introducing current and a way of observing oscillation. In the circuit of figure 1, the RLC series circuit is connected to the function generator, supplying the voltage and current.

The oscillation of charge in the loop can be studied by observing the potential across C with an oscilloscope.

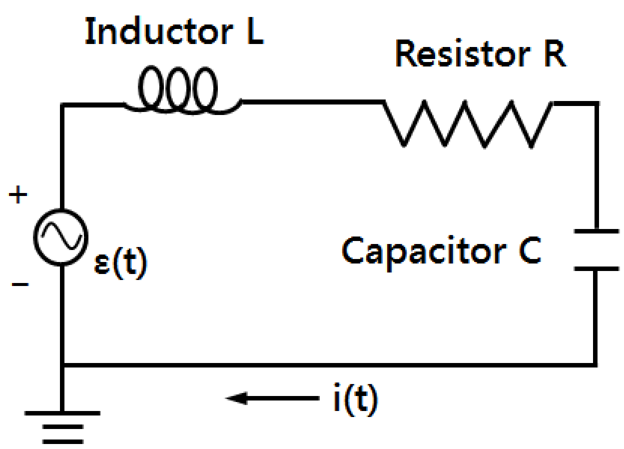

Figure 1. RLC series circuit illustration

From Kirchhoff's rules, the algebraic sum of the changes in potential around a closed loop is zero. Therefore, the equation for this RLC circuit is written as

$$V_R(t)+V_L(t)+V_C(t)=\varepsilon(t) \tag{1}$$

Where $\varepsilon(t)$ is the emf (Electromotive force) supplied by the function generator. $V_R(t)$, $V_L(t)$ and $V_C(t)$ are the potentials across the resistor, the inductor and the capacitor.

We first use the fact that $i(t)$ to obtain

1. Capacitor Characteristic： $C=\dfrac{Q(t)}{V_C(t)}$ & $i(t)=\dfrac{dQ(t)}{dt} \Rightarrow i(t)=C\cdot\dfrac{dV_C(t)}{dt}$
2. Resistor Characteristic (Ohm's Law)： $R=\dfrac{V_R(t)}{i(t)} \Rightarrow V_R(t)=i(t)\cdot R=RC\cdot\dfrac{dV_C(t)}{dt}$
3. Inductor Characteristic： $L=\dfrac{d\Phi(t)}{di(t)}$ & $V_L(t)=\dfrac{d\Phi(t)}{dt}$

$$\Rightarrow V_L(t)=L\cdot\frac{di(t)}{dt}=LC\cdot\frac{d^2V_C(t)}{dt^2}$$

Then we can get the second order differential equation.

$$\frac{d^2V_C(t)}{dt^2}+\frac{R}{L}\frac{dV_C(t)}{dt}+\frac{1}{LC}V_C(t)=\frac{1}{LC}\varepsilon(t) \tag{2}$$

The solution of equation (2) depends on $\varepsilon(t)$, the emf supplied by the function generator. We provide a square wave of low frequency to RLC series circuit.

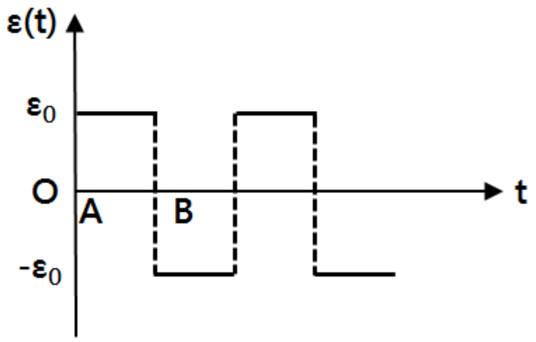

Figure 2. Square wave of $\varepsilon(t)$

The function generator generates a square wave of low frequency, as shown in figure 2. Each times the square wave jumps from + to – or – to +, the change in current sets RLC circuit into oscillation.

1. At point A, $\varepsilon(t)$ jumps from $-\varepsilon_0$ to ε_0. Capacitor is being charged until it is fully charged.
2. At point B, $\varepsilon(t)$ jumps from ε_0 to $-\varepsilon_0$. Capacitor is discharging and then it is reversely charged with a polarity opposite to the original one.

In the following, we discuss these two processes separately：

A. Charging process

$\varepsilon(t) = \varepsilon_0$, the 2nd order differential equation can be represented as

$$\frac{d^2V_C(t)}{dt^2} + \frac{R}{L}\cdot\frac{dV_C(t)}{dt} + \frac{1}{LC}\cdot V_C(t) = \frac{\varepsilon_0}{LC} \tag{3}$$

Consider the initial condition：$V_C(t=0) = -\varepsilon_0$. The solution of equation (3) is

$$\begin{aligned} V_C(t) &= \varepsilon_0\left(1 - 2\sqrt{1+\left(\frac{\beta}{\omega}\right)^2}\, e^{-\beta t}\cos(\omega t - \phi)\right) \\ \Rightarrow V_C(t) &= \varepsilon_0 - 2\varepsilon_0\sqrt{1+\left(\frac{\beta}{\omega}\right)^2}\, e^{-\beta t}\cos(\omega t - \phi) \end{aligned} \tag{4}$$

that

$\beta = \dfrac{R}{2L}$ angular frequency $\omega = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}$ phase $\phi = \tan^{-1}\left(\dfrac{\beta}{\omega}\right)$

amplitude term $|V_C(t)| = 2\varepsilon_0\sqrt{1+\left(\dfrac{\beta}{\omega}\right)^2}\, e^{-\beta t}$ oscillation term $\cos(\omega t - \phi)$

B. Reverse charging process

$\varepsilon(t) = -\varepsilon_0$, the 2nd order differential equation can be represented as

$$\frac{d^2V_C(t)}{dt^2} + \frac{R}{L}\frac{dV_C(t)}{dt} + \frac{1}{LC}V_C(t) = -\frac{\varepsilon_0}{LC}$$

Consider the initial condition: $V_C(t=0)=\varepsilon_0$. The solution of equation (3) is

$$V_C(t)=2\varepsilon_0\sqrt{1+\left(\frac{\beta}{\omega}\right)^2}\,e^{-\beta t}\cos(\omega t-\phi)-\varepsilon_0 \tag{5}$$

that

$\beta=\dfrac{R}{2L}$ angular frequency $\omega=\sqrt{\dfrac{1}{LC}-\left(\dfrac{R}{2L}\right)^2}$ phase $\phi=\tan^{-1}\left(\dfrac{\beta}{\omega}\right)$

amplitude term $|V_C(t)|=2\varepsilon_0\sqrt{1+\left(\dfrac{\beta}{\omega}\right)^2}\,e^{-\beta t}$ oscillation term $\cos(\omega t-\phi)$

By equation (4) and (6) describe the under damped oscillations in RLC circuit, its show in figure 3.

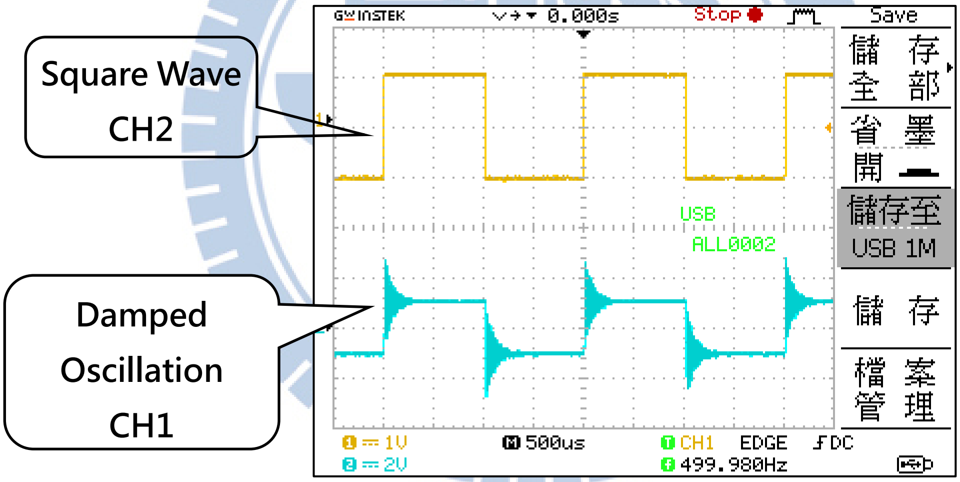

Figure 3. Damped oscillations in oscilloscope

We can see that the amplitude terms of both equation (4) and (5) have the common factor $e^{-\beta t}$. And the oscillation terms have the common factor $cos(\omega t)$.

[Note] $\beta=\dfrac{R}{2L}$ & $\omega=\sqrt{\dfrac{1}{LC}-\left(\dfrac{R}{2L}\right)^2}$

That is, the value of the charge on the capacitor undergoes a damped harmonic oscillation in analogy with a mass–spring system moving in a viscous medium. β and ω depend on R, L, and C. The behavior of RLC circuit is shown as below：

1. When $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, $\omega = 0$ There is no oscillation. It called 「**Critical Damping**」.

2. When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, ω is an imaginary number and $\cos(\omega t) = \cosh(j\omega t)$, which is a Hyperbolic cosine function. There is no oscillation. It called 「**Over Damping**」.

3. When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, ω is a real number. Oscillation occurs. It called 「**Under Damping**」.

   If $\left(\frac{R}{2L}\right)^2 << \frac{1}{LC}$, the angular frequency of the oscillation would approach

   $\omega \approx \sqrt{\frac{1}{LC}} = \omega_0$, called 「**nature angular frequency**」.

In Reverse charging process, the graphs of $V_C(t)$ versus time for these three cases are shown in figure 4.

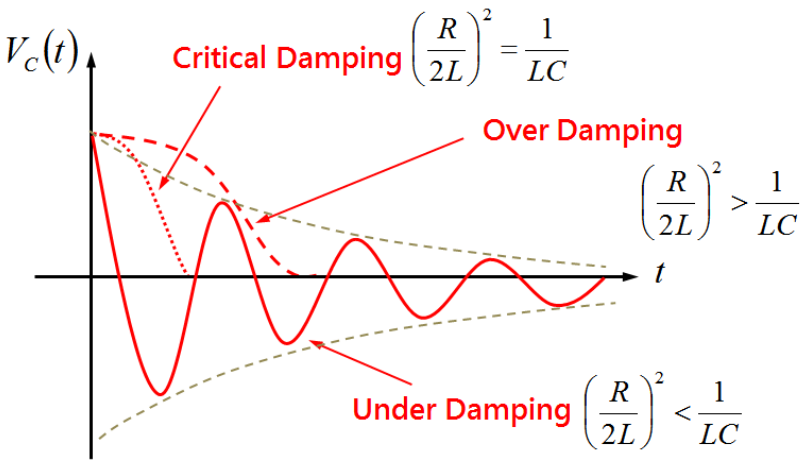

Figure 4. Oscillations phenomenon versus time

Remarks：

1. Make sure that your circuit is not a short circuit before you turn the power on.
2. Make sure that the switch of the resistor, the inductor, and the capacitors are off.

Procedure：

A. Under damping I

1. Set up the apparatus as shown in figure 5.
2. Set R = 800 Ω, L = 10 mH and C = 100 pF.
3. Calculate β.
4. Turn on the function generator. Set the generator to produce a square wave, set the frequency to 500 Hz and output amplitude control to 1.00 V.
   [Note] that CH2 $V_{P-P} = 2.00$ V.
5. Turn on the oscilloscope. Adjust「TIME/DIV」and 「VOLT/DIV」knob until there is 2-3 complete square waves.
   [Note] If not, adjust the scope controls or check the circuit.

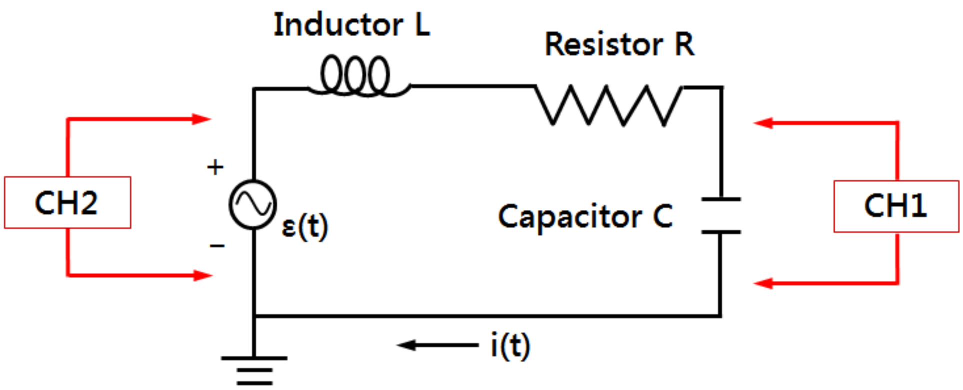

Figure 5. RLC series circuit set-up

6. Record the amplitude $|V_C(t)|$ and it's correspond time t from a complete under damping waveform.
7. Plot $V_C(t)-t$ diagram, analyze the linear regression line to find β and calculate the percentage error.

B. Under damping II

1. Fix R= 100 Ω, L = 10 mH. Change different capacitances (It's suggested that the capacitance should be between 0.002 μF and 0.04 μF)
2. As in step A. Observed the under damping wave again.
3. Calculate angular frequency ω at different capacitance.
4. Find the damped frequency f or period T, and calculate the angular frequency ω.
5. Plot $\omega^2 - \frac{1}{C}$ diagram. Find the line slope, compare it with theoretical value and calculate the percentage error.

C. Critical and over damping

1. Keep the inductance L, capacitance C constant. Vary the resistance R, observe the critical damping and over damping oscillation. Explain what different between the critical damping and over damping oscillation.

Questions：

1. There is a perfect square wave on the Oscilloscope of CH2, before we connect the function generator to the circuit. But after connected to the circuit, there is not a perfect square wave, what happened? Please explain.
2. Given that C = 0.01 μF, L = 10 mH or C = 100 pF, L = 10 mH, what is the values of R if you would observe the under damped oscillation, respectively? Please explain.
3. Please find some applications of damped oscillation in daily life.