Unit-06 RLC Series Circuit Experiment II

Objective：

In this experiment, we study the frequency response of RLC circuit excited by a sinusoidal signal.

Apparatus：

Oscilloscope, function generator, resistor, capacitor, inductor

Principle：

From unit16 RLC circuit oscillations, we can get the second order differential equation.

$$\frac{d^2V_C(t)}{dt^2}+\frac{R}{L}\cdot\frac{dV_C(t)}{dt}+\frac{1}{LC}\cdot V_C(t)=\frac{\varepsilon(t)}{LC} \quad (1)$$

In this experiment, we would study RLC circuit which is driven by a sinusoidal wave generate forced oscillations. Show in figure 1.

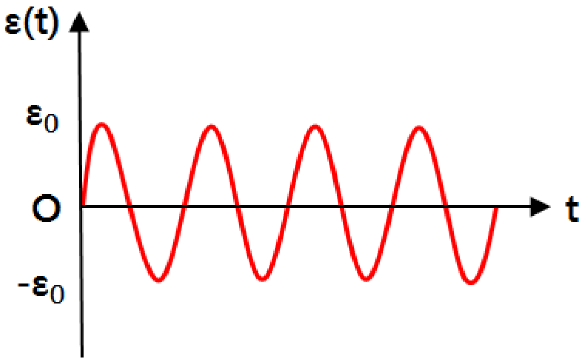

Figure 1. Sine wave of of $\varepsilon(t)$

While the electromotive force $\varepsilon(t)$ is written as $\varepsilon(t)=\varepsilon_0\,cos(\omega t)$, from equation (1), we can get

$$\frac{d^2V_C(t)}{dt^2}+\frac{R}{L}\cdot\frac{dV_C(t)}{dt}+\frac{1}{LC}\cdot V_C(t)=\frac{\varepsilon_0}{LC}\cos(\omega t) \quad (2)$$

The solution of equation (2), can get the $V_R(t)$

$$V_R(t) = \frac{\varepsilon_0 R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \cdot \cos(\omega t - \phi) \qquad (3)$$

that

phase $\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$ oscillation term $\cos(\omega t - \phi)$

Amplitude term $|V_R(t)| = \frac{\varepsilon_0 R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

The amplitude of $|V_R(t)|$ depends on the frequency. A graph of amplitude as a function of frequency will look like curve in figure 2. The amplitude of $|V_R(t)|$ reaches a maximum value ε_0, when $\omega = \sqrt{\frac{1}{LC}}$. This condition defines the resonance angular frequency ω_0, and this phenomenon is called **resonance**

The amplitude of $|V_R(t)|$ to $\frac{\varepsilon_0}{\sqrt{2}}$ at the cutoff frequencies ω_l and ω_k, as shown in figure 2. A bandwidth, a half-width of frequency, is defined as $\Delta\omega = \omega_h - \omega_l$.

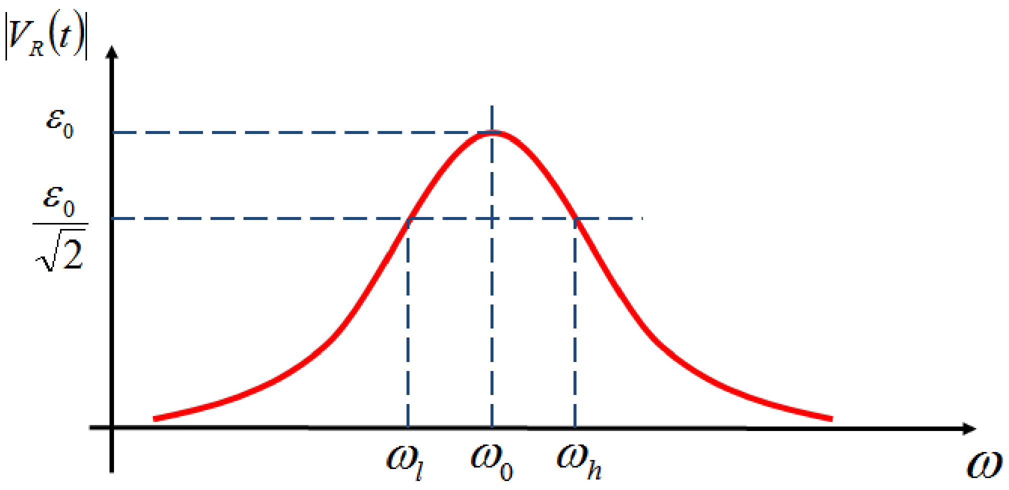

Figure 2. $V_R(t)$ Amplitude versus angular frequency diagram

For RLC series circuit, a half-width of frequency $\Delta\omega = \frac{R}{L}$, it's independent on the capacitance C. Moreover, the resonance angular frequency depends on L and C. The characteristics of RLC circuit vary with the resistance, inductance and capacitance.

Remarks：

1. Make sure that your circuit is not a short circuit before you turn the power on.
2. Make sure that the function generator, oscilloscope, resistor and capacitor are off.

Procedure：

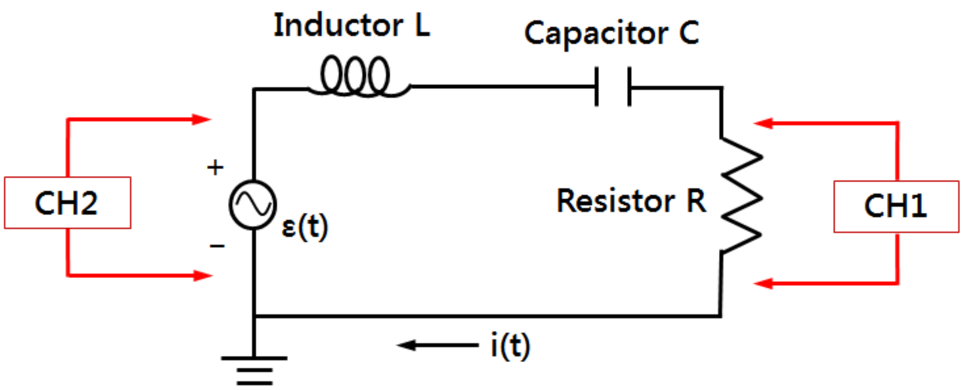

Figure 3. RLC series circuit set-up

1. Set up the apparatus as shown in figure 3.
2. Set R = 1 kΩ, L = 10 mH and C = 0.001 μF.
3. Calculate the resonance angular frequency ω_0 and resonance frequency f_0.

$$\omega_0 = \sqrt{\frac{1}{LC}} \quad \& \quad f_0 = \frac{\omega_0}{2\pi}$$

4. Use the equation (3) to get the theoretical values of $|V_R(t)|$ and input signal phase difference when the frequency is the resonance frequency.
5. Turn on the function generator. Set the generator to produce a sine wave with amplitude of 1.00 V.
   [Note] that CH2 $V_{P-P} = 2.00$ V.
6. Set the output signal's frequency of function generator to resonance frequency f_0.
7. Set the oscilloscope to Lissajous mode.
8. Adjust the frequency until we get the Lissajous pattern which frequency ratio is equal to 1 and the phase difference is zero.

9. Set the oscilloscope to normal mode and Record the frequency, and amplitude $|V_R(t)|$.

10. Vary different frequency several times, record frequency and amplitude $|V_R(t)|$.

11. Plot 作 $|V_R(t)|-\omega$ diagram.

    [Note] It need to including resonance frequency f_0 and cutoff frequency ω_l and ω_h.

12. Used interpolation method to calculate cutoff frequency ω_l and ω_h.
13. Calculate half-width $\Delta\omega$ from this curve and compare with the theoretical value.
14. Keep the inductance L and the capacitance C constant. Vary the resistance R to 2 kΩ, and repeat the above steps.

Questions：

1. Prove the half-width $\Delta\omega = \frac{R}{L}$ for RLC series circuit excited by a sinusoidal signal.
2. What are different graphs with different resistances? Please explain.
3. If we keep the resistance R and the capacitance C constant, and vary the inductance L, how do the resonance frequency and half-width will change with the inductance? Please explain.