

Laboratory Regulation

Before operation :

1. Check the instrument list, and make sure they are OK. If there is something wrong, inform the TA or instructor.
2. No chatting, no playing in the lab. Do not treat the experimental apparatus as a toy ◦
3. Never wear slippers or sandals at the lab.
4. No Food, no drinking except for water.

During operation :

1. Inform the TA or instructor when any instrument does not work properly.
2. Laboratory computer is used only for collecting and analyzing experimental data.
3. Upon finishing the experiment, check your result immediately. If the experimental error is too large, please figure out the reasons and repeat the experiment.
Engineering calculator and software “MS excel” will help in analyzing data.
4. If you have problem, please think by yourself first. Do not rely on the TA too much ◦

After operation :

1. After the TA checks the data and gives the signature, then you can put back everything.
You will get zero point if you miss checking data with the TA.
2. Put back the instruments, and make sure the TA has checked.

Precautions :

1. The use of power :

There are two outlets, 110 V and 220 V in ac voltage, besides experimental table, please check how much voltage the instrument needs, then plug in the proper plug to avoid the damage on instruments.

Good manner in operating experimental equipment to keep it in good condition with a long life. For instance, when DC 3 V is required for experiment, make sure all control knobs are down to minima first. Then plug the power cord and turn on the power supply using the power switch or button. Wait for LED light on and then gradually increase the output voltage and current until the voltage display indicate 3V. To power off, reduce the voltage to zero, turn off the power supply and then unplug the cord.

2. The Laser safety :

LASER (Light Amplification by Stimulated Emission of Radiation) is often applied in a laboratory for its high intensity and coherent characteristics. The power of laser used in this laboratory is less than 5 mW. It is still possible to make eyes injuries since its light intensity is about several hundred times of the sun light. It should be operated carefully.

3. Connection of plug and socket

Wrong wire connections happen to the unobservant persons despite that most connectors are designed with mistake proofing. The wrong connection may influence your measurements, and sometimes damage the plug and socket. Before making electrical connection, take a close look at the pin configurations on both plug and socket and find the correct orientation for connection.

4. How should we do if the apparatus does not work?

Based on experiences, there were more than half of the cases equipment would not work because the power has not been turned on, or forgot to plug the plug of instrument into socket, then the second comes to the equipment assemble, or wiring problems. So when you find that equipment do not work, please do not look for assistant very soon, try to develop the ability of discovering the reason which makes instruments does not work (the debug ability will be helpful to their own future development) by the fore said two points, after you make sure it is not caused by the above reasons by check again, then consider it is the equipment's fault.

Absences in lab courses :

1. For either a sick leave or an official leave, the application with a proof should be filed via the school web. Once it is accepted, you need to contact TA or instructor for the schedule of completing missed experiments.
2. In case that the application cannot be filed in advance, you should talk to instructor in person within a week. Making up the missed experiment is not allowed without a fair reason.

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Introduction

Preface :

The education in college, we not only focus on learning professional knowledge but also cultivating good experimental attitudes and habits. During the class, students can obtain the last knowledge as well as learn how to correctly and carefully use the instruments.

Pursuing the truth is the virtue of science. In the laboratory, we should realize what we are doing exactly and record all original data and that is pursuing the truth. Of course, checking whether the data are reasonable is no doubt important, but it is worthy of paying more attention to how to figure out and analyze the questions. After the experience, we should learn which factors cause the results respectively. We should know what leads to the correct answer and what leads to the wrong one. Those abilities mentioned above will benefit you in the future and that is the purpose of this class.

Take Coulomb's law as example, according to Field and Wave Electromagnetics (2/e) wrote by David K. Cheng: 「The force between two charged bodies, q_1 and q_2 , that are very small in comparison with the distance of separation, R_{12} , is proportional to the product of the charges and inversely proportional to the square of the distance, the direction of the force is along the line connecting the charges. 」 Present it mathematically:

$$\vec{F}_{12} = \hat{R}_{12} k \frac{q_1 q_2}{R_{12}^2}$$

Although it is the result from experiments, there is still a question: How small should the both bodies be to be very small in comparison with the distance of separation, R_{12} ? We should keep in mind that a point charge is an ideal assumption as well as a point mass, but in reality they are absent. We can separate two charged bodies as far as possible to fit "very small in comparison with the distance of separation" so long that the volumes are defined. However, a new question comes. Coulomb force could be too weak compared with external force from environment if the bodies are separated too far, and the data would be not accurate. Moreover, we do the experiment in the limited space. The designers and experimenters should make compromise with the current conditions. From above, inaccuracy is not avoidable.

However, it does not mean experiments are worthless due to inaccuracy. Experiments are essential to examine the truth. The wonderful theory could be abandoned without experimental support.

How can we overcome inaccuracy during the experiment? Practicing and being cautious lead to excellent consequence.

Here are some basic concepts of pursuing the truth during the experiment.

1. All data should be recorded without modification. Even the data are wrong due to clerical error, just draw two lines over them. Remember, keep all row data.
2. Record the data in the form with unit.
3. Single datum is meaningless because it could contain serious error. Therefore every experiment should be recorded five times.

Unit (SI unit) :

We should put a space between value and unit like 3 kg 、 4 K 、 5 cm ...etc.

When a name is use as unit, follow the rule below:

Newton (name) → newton (unit) → N (unit)

Hertz (name) → hertz (unit) → Hz (unit)

Ampere (name) → ampere (unit) → A (unit)

When the unit is not a name, write it in small letter:

meter = m, hour = h, second = s; but liter = liter = l = L is the only exception.

In any measurement, we could not get the exact value even with fine instruments; therefore, estimation, significant figure, and error come.

Estimation is not mentioned since we have already learnt it in high school. Here we only describe rounding, significant figure, and error.

Scientific Notation :

Scientific notation (also referred to as scientific form, standard form or standard index form) is a way of expressing numbers that are too big or too small to be conveniently written in decimal form. It is commonly used by scientists, mathematicians and engineers, in part because it can simplify certain arithmetic operations. On scientific calculators it is known as "SCI" display mode

In scientific notation all numbers are written in the form $m \times 10^n$. (m times ten raised to the power of n)

Most calculators and many computer programs present very large and very small results in scientific notation, typically invoked by a key labelled 「EXP」 or 「E」 (for exponent), depending on vendor and model.

[example 1] 1.632E-19 to 1.632×10^{-19}

[example 2] 3.00E8 to 3.00×10^8

Significant Figure :

23.76 are four significant figures. We can express the length in term of 2.376×10^{-4} km, 0.0002376 km, 0.2376 m, 23.76 cm, 237.6 mm, 237600 m, ...etc. There are all four significant figures in those mentioned above, because those “leading and trailing” 0s are not significant. Therefore, unit conversion does not affect the digit of significant figure.

If 0 does not stand for the decimal point, then boss “zero” and “nonzero” are significant figures. For example, 0.50006 and 34.209 are five significant figures. There are some rules for “0”:

1. Leading zeros are never significant.
2. Any zero contained between two non-zero number is significant.
3. Final or tailing zeros are significant only if it is after a decimal point.
4. Non-zero digits are always significant.

A. Arithmetic of Significant

(a) Addition and Subtraction Rules

If uncertainty exists in any row, then the answer is uncertain after addition and subtraction.

[example 1] $49.57 + 2903.4050 + 9.679 + 5.08 = 2967.7340 \rightarrow 2967.73$

[example 2] $123.579 - 12.41 = 111.169 \rightarrow 111.17$

(b) Division, Multiplication and Square Root Rules

The significant digit of product is determined by less significant digit of dividend, divisor, multiplicand or multiplier.

[example 1] $9500635 \times 0.58 = 5510368.30 \rightarrow 5500000 = 5.5 \times 10^6$

[example 2] $36.94 \times 28.55 = 1054.6370 \rightarrow 1055$

[example 3] $9357.98 \div 508 = 18.421 \rightarrow 18.4$

B. Banker's Rounding

Rounding financial records is extremely important. For accuracy reasons, computations are often performed with more digits than just hundredths of a dollar. The results are then rounded to hundredths of a dollar. This may all seem boring to you, but adding a penny here and there adds up real fast to large financial institutions.

1. If the part of value that will be aborted, when its first digit is equal to or greater than 6, it should round to upward.
[example] $30.29 \rightarrow 30.3$
2. If the part of value that will be aborted, when its first digit is equal to or less than 4, remove the value directly.
[example] $30.24 \rightarrow 30.2$
3. If the part of value that will be aborted is just one digit and the value is 5. Consider the previous digit was odd or even.

- (a) If there are any non-zero digits beyond the thousandths digit, then add one to the hundredths
[example] $30.256 \rightarrow 30.3$
- (b) Now we know the thousandths digit = 5 and all other digits beyond thousandths are 0. The number is exactly between two pennies. We now "round towards even," that is, if the hundredths digit is odd, then add one. Otherwise, the hundredths digit is even, so leave it alone.
[example] $30.350 \rightarrow 30.4$
[example] $30.850 \rightarrow 30.8$

Error Representation :

Error always exists, there is no measurements can get exactly accurate. In order to minimize the errors, so the best way is to know where the error from.

There are three types of errors can be deviated into system error, manual operating error, and random error:

A. Systematic Error

- (a) **Imperfect Calibration of Measurement Instruments:** It comes from improper design or aging of instruments. We should calibrate the instruments before experiment.
- (b) **Changes in Environment:** For example, thermal expansion and contraction will cause error during length measurement; therefore, an air conditioner is a solution.
- (c) **Imperfect Methods of Observation:** It comes from inaccurate theory and improper measurement setup.

B. Random Error

It is always present in a measurement. It is caused by inherently unpredictable fluctuations in the readings of a measurement apparatus or in the experimenter's interpretation of the instrumental reading. The only solution is to increase times and figure the data statistically.

Measurement is limited intrinsically; we must admit that any physics, order, and even empirical formula are not "equal" to the truth we try to express. We just describe it personally. Here we are going to discuss how to express the data statistically.

C. Manual operating Error

It is often caused by carelessness like calculation error and clerical error. The solution is doing the experiment repeatedly (In this class, 5 times are needed and it is better to do it in tern) to find out unreasonable data. Do not falsify the wrong data. Repeats are the only solution and remember do not take the data solely, because the new data does not belong to the original ones. Additionally, poor habits are also the possible reason for the artificial error.

Data Representation :

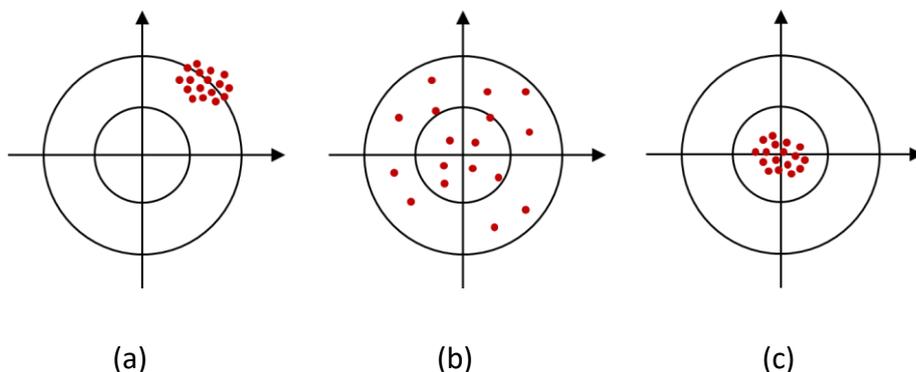


Figure 1. Precision and accuracy illustration

In order to describe precisely physical quantity measured from the experiment, quantity, precision and accuracy, and unit should be included:

- A. Quantity and unit :** Every quantity (estimation included) should be written in form of significant figure and scientific notation ($a.bc \times 10^n$ unit) except for some physically-defined values (such as : $c \sim 3.00 \times 10^8$ m/s , $e \sim 1.60 \times 10^{-19}$ Cetc) and numerical constants (such as : R , N_A etc).
- B. Precision and Accuracy :** Usually expressed in form of “ $a \pm b$ ”. Strictly speaking, precision and accuracy are different.
- (a) **Precision** – It is a measure of how well the result has been determined, without reference to its agreement with the true value, and it also reflects the random error. In figure 1(a), the data are precise and random error is small.
- (b) **Accuracy** –It is a measure of the correctness of the result and it reflects the systematical error. In figure 1(b), the data are accurate and systematical error is small.
- (c) **Precision and Accuracy** – The synthetic index is random error and systematical error. In figure 1 (c) the data are both precise and accurate. Random error and systematical error are both small.

Statistical Analysis :

Statistical analysis is often applied to data analyzing and it is a logistical and powerful instrument.

A. Mean

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \sum_{i=1}^n \frac{x_i}{n}$$

To be notice, mean does not represent the real value and even not the most possible value. We can state it nothing more than “representative”.

B. Deviation

$$d_i = x_i - \bar{x}$$

C. Average deviation

$$D = \frac{\sum_{i=1}^n |d_i|}{n}$$

D. Average deviation percentage

$$\frac{D}{\bar{x}} \times 100\%$$

E. Standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n-1}}$$

When number of measurements is huge (ideally $n \rightarrow \infty$), data can be described by a normal distribution. The deviation of a value can be presented in terms of numbers of average standard deviation and the percentage of data points with such value of the data set is obtained. For a normal distribution, the values less than one standard deviation from the mean account for 68.27% of the set; while two standard deviations from the mean account for 95.45%; and three standard deviations account for 99.73%. For instance, a statement of 「mean mass 127 g with standard deviation of 2 g」 means that 「about 70% of measured values are between 125 g and 129 g」.

F. Standard deviation of the mean

$$\sigma_x = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n(n-1)}}$$

If we do some more measurements, the standard deviation would not change appreciably. On the other hand, the standard deviation of mean would slowly decrease we increase n . However, the factor \sqrt{n} grow rather slowly as we increase n . For example, if

we want to improve our precision by a factor of 10, that means we will have to increase n by a factor of 100. Thus, in practice, if we want to increase our precision appreciably, we will probably do better increase our technique than to rely merely on increased numbers of measurements.

G. Correlation coefficient

$$\gamma = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n\sigma_x\sigma_y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$$

$\gamma = 0$	non-correlation
$0.0 < \gamma < 0.3$	low correlation
$0.3 < \gamma < 0.7$	moderate correlation
$0.7 < \gamma < 1.0$	high correlation
$\gamma = 1$	fully correlation

H. Experimental result

The data should be record in form of $\bar{x} \pm \sigma_x$ with unit.

Mean and Error Transfer :

Two or more experimental results of the arithmetic, should be consider the error transfer.

Let $x = \bar{x} \pm \sigma_x$ and $y = \bar{y} \pm \sigma_y$

A. Error Transfer of Addition and Subtraction

$$\overline{x \pm y} = \bar{x} \pm \bar{y} \quad \text{and} \quad \sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$$

$$x \pm y = (\bar{x} \pm \bar{y}) \pm \sigma_{x+y} \quad \text{and} \quad \sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

In general form :

$$\sigma_N^2 = \sum_{i=1}^n \sigma_i^2$$

B. Error Transfer of Multiplication and Division

(a) Multiplication

$$x \times y = \overline{(xy)} \pm \sigma_{xy}$$

$$\text{where } \overline{(xy)} = \bar{x} \times \bar{y} \quad \text{and} \quad \sigma_{xy} = \sqrt{\left[\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_y}{\bar{y}}\right)^2\right]} \times (\bar{x} \times \bar{y})$$

(b) Division

$$\frac{x}{y} = \overline{\left(\frac{x}{y}\right)} \pm \sigma_{\frac{x}{y}}$$

$$\text{where } \overline{\left(\frac{x}{y}\right)} = \frac{\bar{x}}{\bar{y}} \quad \text{and} \quad \sigma_{\frac{x}{y}} = \sqrt{\left[\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_y}{\bar{y}}\right)^2\right]} \times \frac{\bar{x}}{\bar{y}}$$

In general form :

$$\left(\frac{\sigma_N}{y}\right)^2 = \left(\frac{\sigma_1}{y_1}\right)^2 + \left(\frac{\sigma_2}{y_2}\right)^2 + \dots + \left(\frac{\sigma_n}{y_n}\right)^2$$

where y is the average derived and y_1, y_2, \dots, y_n are average of every element in the calculation.

C. Error Transfer of Power law

$$x^l \times y^m = (\bar{x}^l \times \bar{y}^m) \pm \sigma_{x^l y^m}$$

$$\text{where } \overline{x^l \times y^m} = (\bar{x}^l \times \bar{y}^m) \quad \text{and} \quad \left(\frac{\sigma_{x^l y^m}}{\bar{x}^l \times \bar{y}^m}\right)^2 = l^2 \left(\frac{\sigma_x}{\bar{x}}\right)^2 + m^2 \left(\frac{\sigma_y}{\bar{y}}\right)^2$$

D. Error Transfer of General Standard Deviation

Set $N = f(x, y)$ and then,

$$\sigma_N = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2}$$

Least Square Regression Analysis :

It is an analysis instrument in common use. By fitting the optimized regression, we are able to minimize the sum of square of vertical distance to the regression.

For n data sets:

$$(x_i, y_i), \quad i = 1, 2, 3, \dots, n$$

A. Linear regression

If the optimized regression is the form of $y = f(x) = Ax + B$, where A and B unknown, then the sum of square of vertical distance to the regression is:

$$D(A, B) = \sum_{i=1}^n [f(x_i) - y_i]^2 = \sum_{i=1}^n [Ax_i + B - y_i]^2$$

The so-called optimization is to decrease $D(A, B)$ for a minimum,

$$\frac{\partial D}{\partial A} = 0 \quad ; \quad \frac{\partial D}{\partial B} = 0$$

$$\begin{aligned} \frac{\partial D}{\partial A} &= \sum_{i=1}^n \frac{\partial}{\partial A} [Ax_i + B - y_i]^2 = \sum_{i=1}^n 2[Ax_i + B - y_i]x_i \\ &= 2 \sum_{i=1}^n [Ax_i^2 + Bx_i - x_i y_i] = 2 \left[A \sum_{i=1}^n x_i^2 + B \sum_{i=1}^n x_i - \sum_{i=1}^n x_i y_i \right] = 0 \\ \frac{\partial D}{\partial B} &= \sum_{i=1}^n \frac{\partial}{\partial B} [Ax_i + B - y_i]^2 = 2 \left[\sum_{i=1}^n Ax_i + \sum_{i=1}^n B - \sum_{i=1}^n y_i \right] \\ &= 2 \left[A \sum_{i=1}^n x_i + nB - \sum_{i=1}^n y_i \right] = 0 \end{aligned}$$

And the solutions are

$$A = \frac{n[\sum_{i=1}^n x_i y_i] - [\sum_{i=1}^n x_i] \cdot [\sum_{i=1}^n y_i]}{n[\sum_{i=1}^n x_i^2] - [\sum_{i=1}^n x_i]^2} \quad ; \quad B = \frac{[\sum_{i=1}^n y_i] - A[\sum_{i=1}^n x_i]}{n}$$

We can just input (x_i, y_i) to engineering calculator or software like MS-excel, and then A and B can be obtained.

B. Exponential regression

It describes the data in form of $y = A \times e^{Bx}$. Also, it can be applied through calculator or software.

C. Logarithmic regression

It describes the data in form of $y = A \times \ln x + B$, Also, it can be applied through calculator or software.

D. Power regression

It describes the data in form of $y = A \times x^B$, Also, it can be applied through calculator or software.

Format of a laboratory report

Cover (Title page)

Free style

Indicate the title and your name, department, class, group, and student ID number.

Introduction

State the objective of the experiment and provide the reader with background to the experiment.

Background stories, previous researches, and interesting related information

Theories

Briefly provide the concept and related theories.

Explain how you derived that hypothesis.

Methods

Document the methods, subjects, materials, and equipment you used to collect data.

Results

Present the results of the clinical studies in a clear and concise manner. Data may be offered in tables, charts, and graphs.

Graphs and figures must both be labeled with a descriptive title. Label the axes on a graph, being sure to include units of measurement.

Discussion

Interpret the results and explain how they relate to the theoretical prediction.

Compare your **experimental results** and **the theoretical prediction** and analyze how they consistent or inconsistent.

Check errors in experimental data and find out the sources of them.

Discuss any limitations or sources of error in the experiment and how they can be improved upon.

Conclusion

Summarize the main findings of the experiment and the conclusions that can be drawn

Avenue suggestions for future research

Any related application in daily life?

References

Cite all sources in the report. Name of the author, title of the article, name of the journal with volume and page numbers, and year of publication should be included.

Format of a figure

Example :

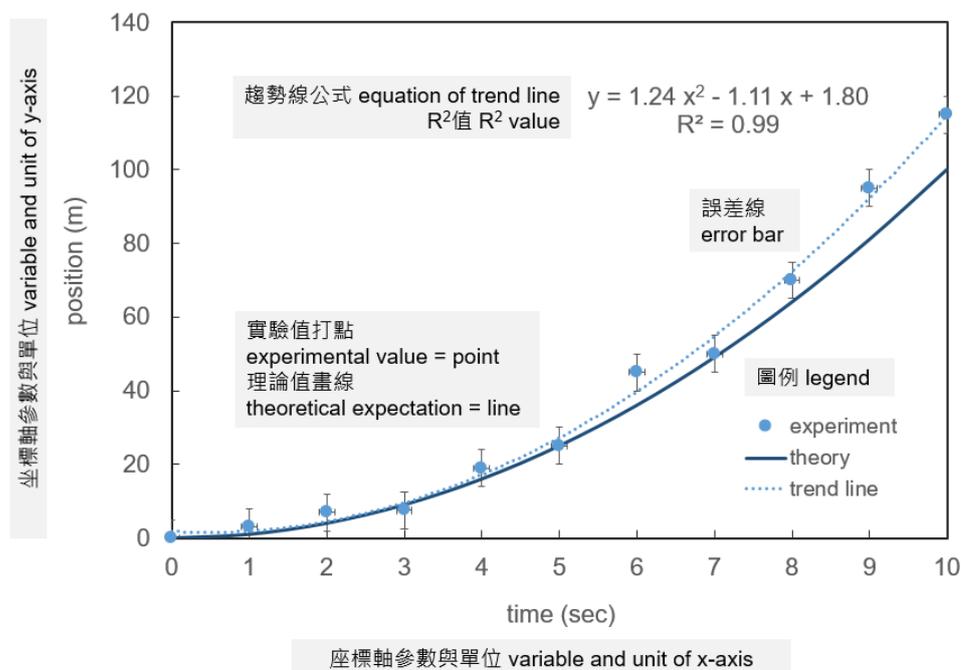


Figure 1. Plot of position of an object vs. time. Circles are experimental data and the solid line is obtained by the theory. The dashed line is the trend line. (Figure caption)

Constitutes :

- Variables with units of axes
- data : points (x_i, y_i)
- error of each experimental value : error bar
- theory : a solid line or curve
- trend line : a dashed line or curve
- legend : indication of the meaning of each data set
- Equation of trend line $y(x)$ and R^2 value (a measure about the goodness of fit of $y(x)$)

$$R^2 = 1 - \frac{\sum(y_i - y(x_i))^2}{\sum(y_i - \bar{y})^2} \quad \text{and} \quad \bar{y} = \frac{\sum y_i}{n} \quad n: \text{number of data points}$$

- Figure caption : figure number and description

The rules to calculate significant figures for arithmetic

Significant figures express the uncertainty of a measurement or number. All measurements have some degree of uncertainty in their value. This is inherent in measuring tools and variations between people taking measurements. For instance, using a ruler to measure the length of a pencil. The graduation of the ruler is 1 mm. The measured value is 150.51 mm where 150.5 are precise and 「1」 on the right of 「5」 is estimated. 150.51 mm has five significant figures. For another example, using two electronic scales to measure the same weight. 2.20 g and 2.1745 g are obtained, respectively. Hence, we know that the accuracy of these two scales are ± 0.01 g and ± 0.0001 g, respectively. Notice that 0 in 「2.20 g」 should not be omitted to indicate ± 0.01 g in accuracy. Upon measurement, your significant figures should include all measurable digits (the digits that correspond to the marks on the ruler) as well as one estimated position beyond the smallest measurable digit.

It may be easy to handle to use scientific notation for the measured value in calculations. In scientific notation all numbers are written in the form $m \times 10^n$. (m times ten raised to the power of n). For instances,

$$0.3456 = 3.456 \times 10^{-1}$$

$$4567.8 = 4.5678 \times 10^3$$

where the **bold numbers** are precise and the grey number with underline are estimated

In the experiment, some values – like averages or totals – are not measured directly. Rather, they are calculated from measured values. In a calculation, the uncertainty of the result is determined by the uncertainty of the measurements. In this case, the calculated values must also follow the rules of significant figures, and the number of significant figures in the calculated value is dependent on the values used in calculation.

Addition/Subtraction :

- Numbers of same significant figures (straightforward)

$$3.456 \times 10^{-1} + 1.001 \times 10^{-1} = 4.457 \times 10^{-1}$$

All numbers have four significant figures.

- Numbers of different significant figures

$$3.45\bar{6} \times 10^{-1} + 1.\bar{1} \times 10^{-1} = 4.\bar{5} \times 10^{-1}$$

The number of decimal places in the result should equal the smallest number of decimal places in any term in the sum.

- a **carry** (a digit that is transferred from one column of digits to another column of more significant digits)

$$3.45\bar{6} \times 10^{-1} + 8.00\bar{1} \times 10^{-1} = 11.45\bar{7} \times 10^{-1}$$

Both $11.4\bar{6} \times 10^{-1}$ (four significant figures) and $11.45\bar{7} \times 10^{-1}$ (five significant figures) are accepted.

Multiplication/Division :

- Numbers of same significant figures (straightforward)

$$(3.45\bar{6} \times 10^{-1}) \times (1.00\bar{1} \times 10^{-1}) = 3.45\bar{9} \times 10^{-2}$$

All numbers have four significant figures

- Numbers of different significant figure

$$(1.00\bar{1} \times 10^7) \times (1.\bar{1} \times 10^{-1}) = 1.\bar{1} \times 10^6$$

The number of significant figures is the same as the number of significant figures in the quantity having the lowest number of significant figures.

- a **carry** (a digit that is transferred from one column of digits to another column of more significant digits)

$$(5.00\bar{1} \times 10^2) \times (5.00\bar{1} \times 10^7) = (25.01\bar{0} \times 10^9)$$

Both $25.0\bar{1} \times 10^{-1}$ (four significant figures) and $25.01\bar{0} \times 10^{-1}$ (five significant figures) are accepted.

Basic measurements and Statistics I

Objective :

1. Use Vernier caliper to measure the length of an object, inner/ outer diameter and depth of a cup.
2. Use screw micrometer to measure the diameter of a metal wire, and the thickness of a piece of object.
3. Use traveling micrometer to measure the width of a single-slit and double-slit.

Apparatus :

Vernier caliper, screw micrometer, traveling micrometer, cup, thin metal wire, single-slit, double-slit, slit holder, LED

Principle :

In this experiment, we would learn how to use about Vernier caliper, screw micrometer, and traveling micrometer, it was more precisely than ordinary ruler. All these three apparatuses include main-meter and sub-meter.

The deviation comes into existence when these apparatuses use for a long time, it means it cannot reset to zero, and it occurs on screw micrometer frequently. Here, we call that is zero error, and it should always consider when we use any measurement apparatuses.

$$\text{Length of object} = \text{main-ruler reading} + \text{sub-ruler reading} - \text{zero error}$$

A. Vernier Caliper

1. **The construction :** Vernier caliper of 0.05 mm in accuracy is shown in Fig.1. F is the main-ruler and G is the sub-ruler that is attached on main-ruler and able to slide. Pincers A and B are used for measuring the outer diameter of a cup; and pincers C and D are for measuring of the inner diameter of a cup. H is used for measuring the depth of a container. Nut E can fasten the sub-ruler on the main-ruler.

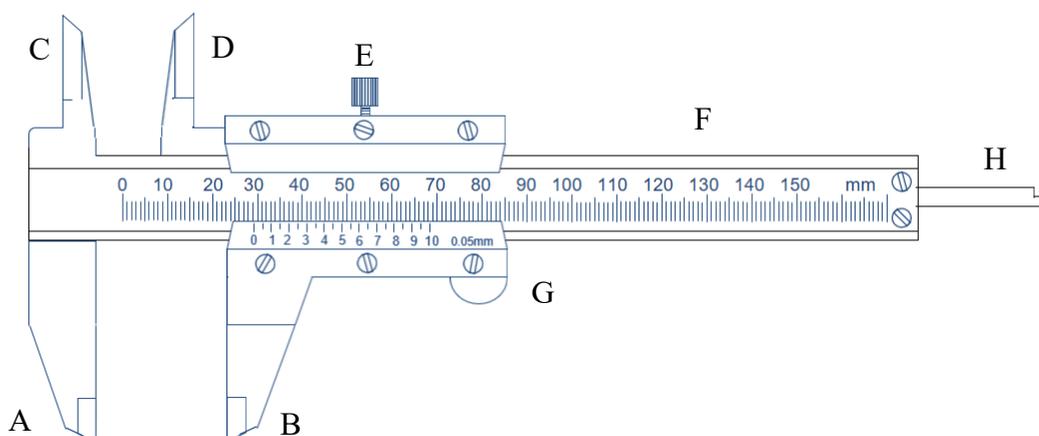


Figure 1. Schematics of Vernier caliper.

- 2. The Accuracy :** continued, Vernier caliper of 0.05 mm in accuracy for example
- the least graduation of the main-ruler is 1 mm.
 - Length of 20 ticks on the sub-ruler as labeled on the main-ruler is equal to 39 mm, which means the length per tick on the sub-ruler is equal to 1.95 mm. The difference between that value and two ticks (2 mm) on the main-ruler is 0.05 mm, and we utilize this feature for a more accurate measurement to 0.05 mm.

3. The Measurement :

Length of object = main-ruler readings + sub-ruler readings – zero error

- Zero error : When never grip an object, main-ruler and sub-ruler will be in alignment at the graduation of zero. If not, we should record the zero error value a_0 .
[Note] When sub-ruler's zero on the main-ruler's zero right, that is positive; on the other hand, that is negative.
- Main-ruler reading : Sub-ruler's zero falls between N mm and N+1 mm, the readings record N mm.
- Sub-ruler reading : Find out the ticks on sub-ruler which aligns to one of the ticks on main-ruler, and multiply it by the accuracy value.

As shown in Fig.2, for example, the scale 11 on sub-ruler aligns the scale 29 on main-ruler, so the readings could record $11 \times 0.05 \text{ mm} = 0.55 \text{ mm}$. If zero error is 0.00 mm, the length is measured as $7 \text{ mm} + 0.55 \text{ mm} - 0.00 \text{ mm} = 7.55 \text{ mm}$.

[Note] Significant figures below two digits of mm, and no estimate value.

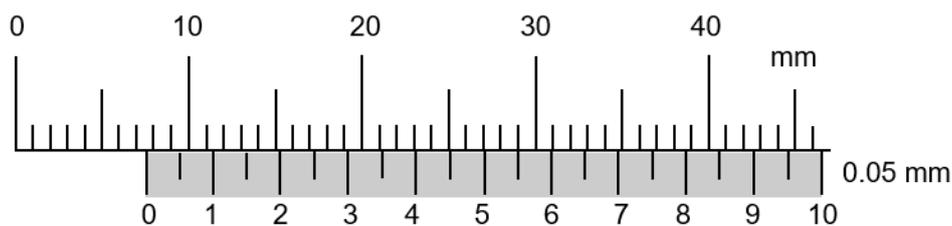


Figure 2. Example for Vernier caliper.

B. Screw Micrometer

- The construction :** Screw micrometer can get a more accurate measurement of the thickness of an object. The structure of a screw micrometer is shown in Fig.3. **C** is main-meter and **D** is sub-meter that is attached on rough thimble **E**. **A** is a fixed end, and the spindle **B** is connected with thimble. **F** is lock nut that fix thimble on main-meter. Screw the rough thimble and clamp the object between **A** and **B**, and then screw fine thimble **H** slightly until you hear three clicks.

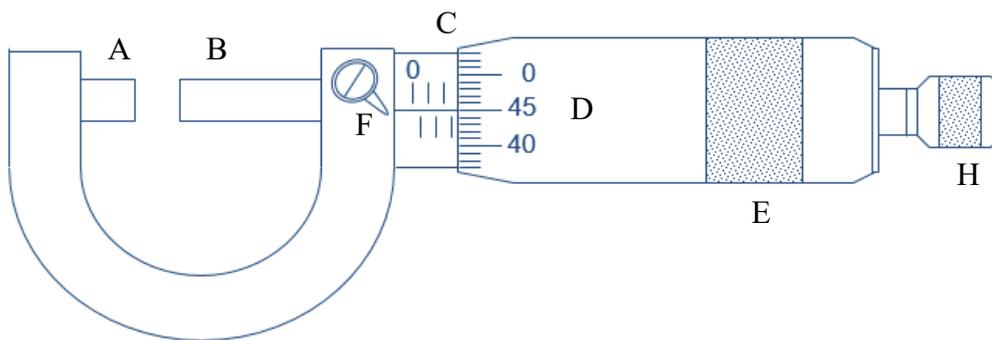


Figure 3. Schematics of Screw micrometer.

2. **The Accuracy** : At a general screw micrometer, the main-meter is divided into two parts about up and down, due to the horizontal line. Whatever up or down, one tick is 1 mm, and the adjacent graduation between up and down is 0.5 mm. On the periphery of sub-meter that the left-end is thimble are 50 ticks. When thimble is screwed one round, main-meter shifts 0.5 mm; it means per tick on sub-meter is 0.01 mm in accuracy.

3. **The Measurement** :

$$\text{Length of object} = \text{main-ruler readings} + \text{sub-ruler readings} - \text{zero error}$$

- (a) Zero error : Before grip an object, main-meter and sub-meter will be in alignment at the graduation of zero. If not, we should record the zero error value a_0 .
 [Note] When the main-meter's horizontal line is under the sub-meter's zero that is positive; on the other hand, that is negative.
- (b) Main-ruler reading : Find out the ticks on main-meter which the edge of sub-meter falls between, as in Fig.3, for example, the readings could record 2.5 mm.
- (c) Sub-ruler reading : Find out the ticks on sub-meter which the main-meter's horizontal line is extended to, it should include estimate value.

As shown in Fig.3, for example, the scale 45.0 on sub-meter corresponding main-meter's horizontal line, so the readings could record $45.0 \times 0.01 \text{ mm} = 0.450 \text{ mm}$. If zero error is 0 mm, the thickness is measured as $2.5 \text{ mm} + 0.450 \text{ mm} - 0 \text{ mm} = 2.950 \text{ mm}$.

[Note] Significant figures below three digits of mm, and the third digit is estimate value.

C. Traveling Micrometer

1. **The construction** : The structure of a traveling micrometer is shown in Fig.4. A is a platform with the set of vertical Vernier caliper B and the set of horizontal Vernier caliper C on it. D is a low-power microscope. The set of Vernier caliper B are used for horizontal measurement; and the set of Vernier caliper C are used for vertical measurement.

2. The Accuracy : same as Vernier caliper

- (a) The least graduation of the main-ruler is 0.5 mm.
- (b) Before grip an object, align the zero grid of main-ruler with the zero grid of the sub-ruler. We fund that the 50th grid of the sub-ruler will align with 49th grid of the main-ruler. It means that the former aligns with 24.5 mm on the latter. Per tick on the sub-ruler is 0.49 mm. The difference between that value and one tick (0.5 mm) on the main-ruler is 0.01 mm, and we utilize this feature for a more accurate measurement to 0.01 mm.

3. The Measurement :

Length of object = main-ruler readings + sub-ruler readings – zero error

- (a) Zero error : Before fasten an object, main-ruler and sub-ruler will be in alignment at the graduation of zero. If not, we should record the zero error value a_0 .
- (b) Both main-meter and sub-meter readings are the same as Vernier caliper.

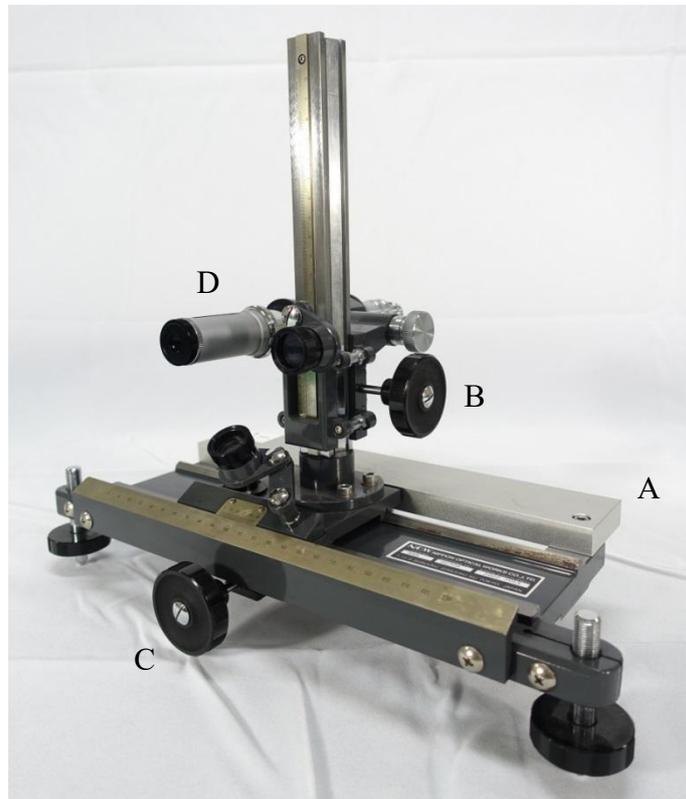


Figure 4. Schematics of Traveling micrometer.

Remarks :

- 1. When the experiments are done, separate the contacts of the screw micrometer to avoid damage.
- 2. Do not over fasten the objects with Verner caliper or screw micrometer. Then screw fine thimble **H** slightly until you hear three clicks.

Procedure :

A. Use of Vernier caliper measure the inner diameter, outer diameter and depth of a cup; and then calculate its capacity.

1. Record the zero error a_0 of Vernier caliper.
2. Clamp the outer periphery of a cup with **A** and **B**, and measure the outer diameter $2R$.
3. Put pincers **C** and **D** inside a cup and clamp it. Read the recorded value of the inner diameter $2r$.
4. Measure the depth h of the cup with **H**.
5. Get the average value and the standard deviation of the mean.
6. Calculate the capacity of the cup. (including the error transfer)

[Note] Capacity of the cup $V = \pi r^2 h = \bar{V} \pm \sigma_V$

$$\bar{V} = \pi \bar{r}^2 \bar{h} \quad \text{and} \quad \sigma_V = \sqrt{\sigma_r^2 (2\pi \bar{r} \bar{h})^2 + \sigma_h^2 (\pi \bar{r}^2)^2}$$

B. Use of screw micrometer measure the diameter of a metal wire, and the thickness of an item.

1. Record the zero error a_0 of screw micrometer.
2. Screw the rough thimble and clamp the thin metal wire or optional object between **A** and **B**.
3. Measure the diameter of thin metal wire and thickness of optional object respectively.
4. Calculate the average value and the standard deviation of the mean respectively.

C. Use of traveling micrometer measure the width of single slit, double-slit, and the central dark fringe of double-slit.

1. Adjust the horizontal level.
2. Fasten the object on the platform, and adjust its position to get a clear image appears in the eyepiece.
3. For an example of single-slit, the definitions of a and b are shown in the Fig.5. Adjust horizontal knob on **C** to make the cross in the eyepiece overlap one end of the item under measurement. Put down the recorded position x_1 . Repeat the procedure and record the position x_2 on the other end. Then the length of the object will be $\Delta x = |x_2 - x_1|$.
4. Measuring the width of a double-slit and the center of dark fringe by the same way.
5. Calculate the average value and the standard deviation of the mean respectively.

[Note] Keep the edges of slit perpendicular to the micrometer.

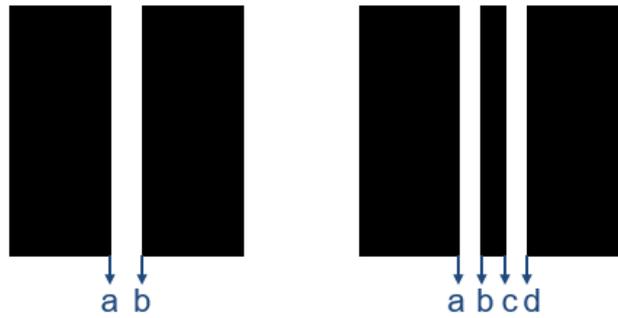


Figure 5. Schematics of single slit (left) and double slit (right).

Discussion hints :

1. When measuring the inner, outer diameter and depth of a cup, you would find that the recorded value differs when you measure with different position. What are the difference between measurements at the same position and measurements at different positions?
2. According to characteristics of the screw micrometer, what is the restriction about the sample we want to measure? Please elaborate on it.
3. In the case that the magnification power of traveling micrometer is changed, does the accuracy be affected of the experiment? Please elaborate on it.
4. How to judge whether the single-slit and double-slit is perpendicular to traveling micrometer or not? If they are not perpendicular, what will the effect be? Please elaborate on it.

Basic measurements and Statistics II

Objective :

Use spherometer to measure the curvature radius of a spherical surface.

Apparatus :

Spherometer, traditional spherometer, plate glass, convex glass

Principle :

A. The Construction :

A spherometer is shown as Fig.1. **A** is a tripod with three arms **B**, **C**, and **D** with the equivalent length. The included angle of each two arms is 120° . **E** is the main-meter, one tick is 1 mm. And there are 100 ticks on the periphery of sub-meter **F**. Probe **H** shifts 1 mm when sub-meter is screwed one round, which means the length per tick is 0.01 mm. We can get the height of probe from the main-meter and sub-meter.

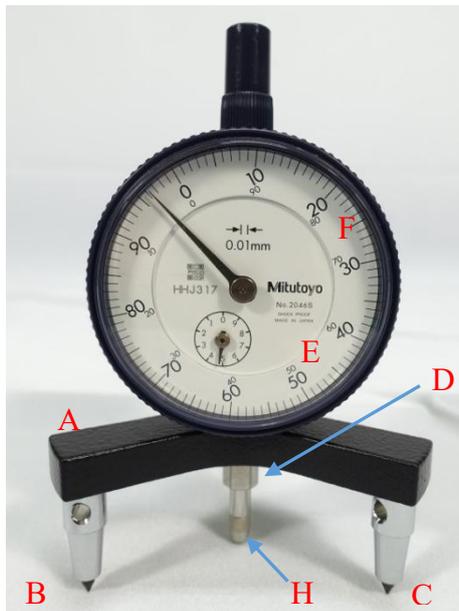


Figure 1. A spherometer.

B. The Measurement :

- Main-ruler reading : Sub-ruler's pointer between N mm and $N+1$ mm, the readings record N mm.
- Sub-ruler reading : Find out the ticks on sub-meter. It should include estimate value.

C. Curvature Radius

As shown in Fig.2, the distance between each arm of the tripod is S , hence **B**, **C**, and **D** form an equilateral triangle. We can make a circumscribed circle of radius r . The extended line of probe **H** is sure to pass the center of the sphere O' . We assume that the line $\overline{O'H}$ intersects plane BCD at O . Let curvature radius is R , if we measure the height is a by probe **H**, and then we could calculate curvature radius R by Pythagorean proposition.

$$r = \frac{S}{\sqrt{3}}$$
$$R^2 = (R - a)^2 + r^2$$
$$R = \frac{S^2}{6a} + \frac{a}{2}$$

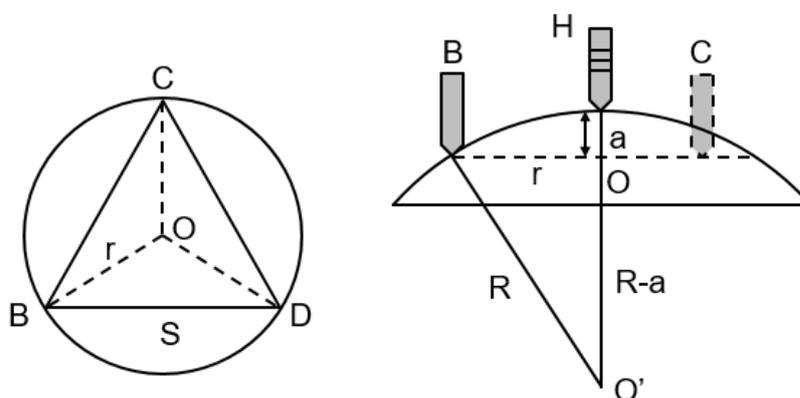


Figure 2. Top view (left) and Side view (right) of a spherometer.

Remarks :

1. When you put spherometer on the glossy glass plate, please gently lay down in prevent from bending the tips.
2. The four arms of the spherometer should contact on the glasses during the period of the experiment.

Procedure :

1. Adjust the fixed tips to make it into an ideal triangle.
2. Push gently down 3 tips **B**, **C**, and **D** on the paper.
3. Remove the spherometer and draw an ideal triangle regarding tip positions **B**, **C**, and **D** on the paper.
4. Use Vernier caliper to measure the length between two fixed tips S °

5. Put the spherometer on the plate glass, and make the tips of **BCDH** contact the plate glass. Record the value a_0 .
6. Put the spherometer on the convex glass, and making the tips of **BCDH** contact the convex glass. Recording the value a_1 .
7. The difference between a_0 and a_1 is a , that is the height of **H** from plane **BCD**.

$$a = |a_1 - a_0|$$

8. Get the average value and standard deviation of the mean.
9. Calculate the curvature radius R . (including the error transfer)

[Note] Curvature Radius $R = \frac{s^2}{6a} + \frac{a}{2} = \bar{R} \pm \sigma_R$

$$\bar{R} = \frac{\bar{s}^2}{6\bar{a}} + \frac{\bar{a}}{2}$$

$$\sigma_R = \sqrt{\sigma_s^2 \left(\frac{\bar{s}}{3\bar{a}}\right)^2 + \sigma_a^2 \left(-\frac{\bar{s}^2}{6\bar{a}^2} + \frac{1}{2}\right)^2}$$

10. Used traditional spherometer to do this experiment.

Discussion hints :

1. The construction of traditional spherometer and spherometer are the same? Please elaborate on it.
2. Can the radius of the convex lens be measured by spherometer? Please elaborate on it.

Linear motion

Objective :

Place the trolley on the nearly frictionless aluminum trajectory for constant acceleration motion to verify the Newton's laws of motion

Apparatus :

Aluminum track, trolley, photogate with pulley, weight, hook, level, string, ruler, yellow cushion pad, electronic scales, track rod clamp, rod stand, digital adaptor(blue · PS-2159), Air-Link(white · PS-3200), computer (software Capstone).

Principle :

Newton's second law of motion defines that the net force acting on an object in motion is equal to the mass of the object multiplied by the acceleration of the object. The equation can be represented as

$$\vec{F} = M\vec{a} \quad (1)$$

where \vec{F} is the vector sum of all forces acting on the object with magnitude and direction, M is the mass of the object, and \vec{a} is the acceleration of the object.

Fig. 1 is the schematic diagram of experimental set-up of Newton's laws of motion. String rounds over a pulley with one end connected to the trolley horizontally and the other end connected to the hook with weights vertically. The hook with weights fall ($-y$ direction) and the trolley moves toward pulley ($+x$ direction) by the string.

m_1 is the total mass of the trolley and the weights on the trolley. The trolley and the trajectory are nearly frictionless. m_2 is the total mass of the hook and the weights on the hook. m_{eff} is the effective mass caused by the spinning of the pulley.

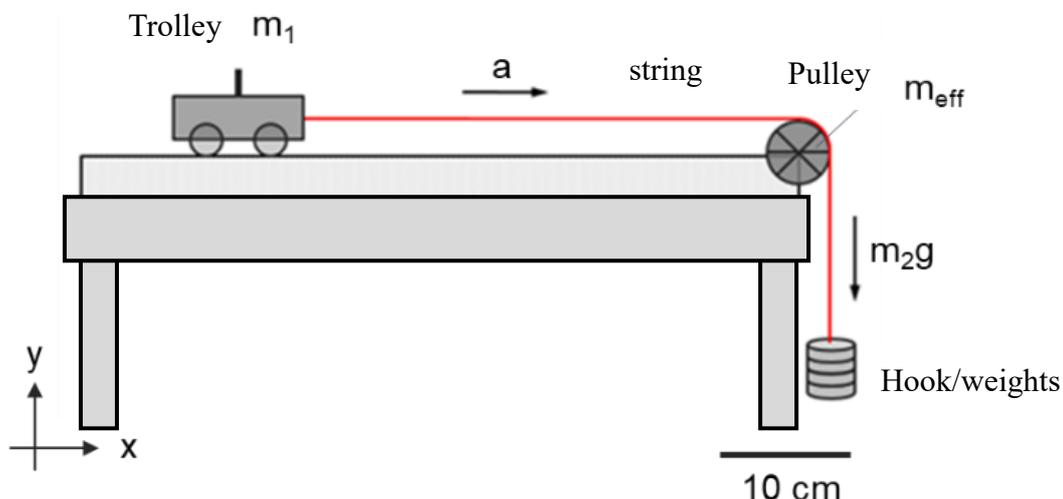


Figure 1. The schematic diagram of experimental set-up of Newton's laws of motion.

In terms of energy point of view, the gravitational potential energy is provided by falling of the hanging objects and mostly transformed to the kinetic energies of trolley with loaded weights and the hanging objects. However, few energies would be dissipated by friction. Moreover, the rotation kinetic energy of pulley also should be taken into accounted. Rotational energy of an object is determined by its rotational inertia I and angular speed ω .

$K_{rotation} = \frac{1}{2}I\omega^2$ usually can be written as $\frac{1}{2}\beta m_r v^2$. β , a dimensionless coefficient, is determined by the mass distribution (m_r) relative to the rotational axis. Therefore, we define the effective mass of the pulley as m_{eff} , the corresponding force to the system due to the energy change is m_{eff} multiplied by $-\vec{a}$. In this experiment, m_{eff} is 4.50 g and the influence of the spinning of pulley can treated as the additional mass of 4.50 g in the system.

According to Newton's second law of motion Equation (1) can be written as

$$F = Ma$$

$$\Rightarrow m_2 g - m_{eff} a = (m_1 + m_2) a$$

$$\Rightarrow m_2 g = (m_1 + m_2 + m_{eff}) a$$

Therefore, the acceleration of the system

$$a = \frac{m_2 g - m_1 g \sin \theta}{m_1 + m_2 + m_{eff}}$$

Software capstone combined with photogate measures the angular speed of pulley and calculate the displacement and linear velocity of the trolley. Fig.2 is the plot of the linear velocity versus time of the trolley. Get the slope of linear fit to data by linear regression. The value is the acceleration of the system a .

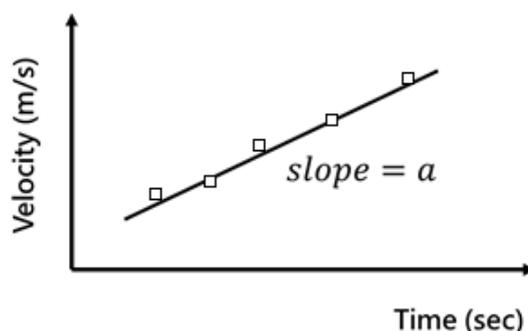


Figure 2. Plot of velocity vs. time.

Figure 3 is the schematic diagram of the set-up of Atwood's machine experiment. String rounds over a pulley with one end connected to the trolley on the inclined plane and the other end connected to the hook with weights vertically. The hook with weights fall ($-y$ direction) and the trolley moves toward pulley (climb up) by the string. Same consideration as above. m_1 is the total mass of the trolley and the weights on the trolley. The trolley and the trajectory are nearly frictionless. m_2 is the total mass of the hook and the weights on the hook. m_{eff} is the effective mass caused by the spinning of the pulley.

According to Newton's second law of motion Equation (1) can be written as

$$F = Ma$$

$$m_2g - m_1g \sin \theta - m_{\text{eff}}a = (m_1 + m_2)a$$

Therefore, the acceleration of the system

$$a = \frac{m_2g - m_1g \sin \theta}{m_1 + m_2 + m_{\text{eff}}}$$

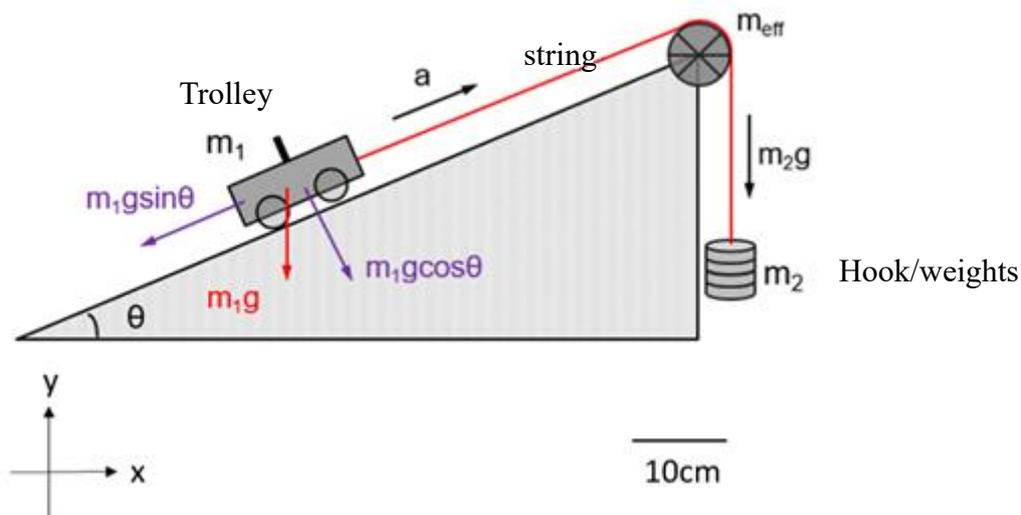


Figure 3. The schematic diagram of the set-up of Atwood's machine experiment.

When $\theta < 10^\circ$ 時, $\sin \theta$ (deg) $\approx \theta$ (rad), acceleration a is in approximation as

$$a \approx \frac{m_2g - m_1g\theta}{m_1 + m_2 + m_{\text{eff}}}$$

meanwhile if $m_1 \gg m_2$ and $m_1 \gg m_{\text{eff}}$, acceleration a can be farther simplified as

$$a \approx \frac{m_2g}{m_1} - g\theta$$

Various pulling strength on the trolley can be obtained by adjust the incline angle. Here, we investigate the influence of the pulling strength on the acceleration of the trolley when it climbs up on the track.

Remarks :

1. Put the level on the trajectory. Adjust the screws on both sides underneath the trajectory to level it. Put cushion on the other end of the trajectory.
2. Pulley and hook are fragile. The weights on the hook should be less than 100 g.
3. Yellow cushion pad should be on the floor right underneath of the hook and weights to prevent them from impacting the floor intensively.

Procedure :

➤ **Motion on the horizontal plane :**

Preparation

1. Put the level on the track. Adjust the screws on both sides underneath the track to level it.
2. Place the trolley on the track. Observe whether the trolley slides toward one side or another of the track. If so, please level the track again until the trolley stops sliding. However, if it cannot be completed, estimate the effect to the experiment.
3. Take an appropriate length of string. Tie one end of the string on O-ring and put the O-ring on the trolley's post. Tie the other end on the hook.
4. Place the string on the Smart Pulley's groove and adjust the position of the O-ring. Make the string paralleled to the track.
5. Put yellow cushion pad on the floor right underneath of the hook and weights.

A. Keep the system's force F constant, and change the system's total mass M

1. Weigh the hook and the weights on the hook using electronic scale and record the value as m_2 .
2. Weigh the trolley with loaded weights using electronic scale and record the value as m_1 .
3. Calculate the total mass $M = m_1 + m_2 + m_{\text{eff}}$.
4. Open software capstone and set all parameters in on the entry screen based on the operation manual.
5. Hold the trolley on the tail end of the track. Click "Record" to start capturing data and release the trolley. Then you can acquire the velocity-time diagram of the trolley. The slope of the diagram is the acceleration a . Repeat steps above for numerous times. Then, record the experimental value of acceleration as $a = \bar{a} \pm \sigma_a$.
6. Keep constant the total mass of the hook and the weights on the hook. Change the total mass of the trolley and the weights (i.e. changing the system's total mass). Repeat above steps.
7. Plot $\frac{1}{a}$ vs. M . Make comparisons with the expected results.

B. Keep the system's total mass M constant, and change the system's force F

1. Weigh the hook and the weights on the hook using electronic scale and record the value as m_2 .
2. Weigh the trolley with loaded weights using electronic scale and record the value as m_1 .
3. Calculate the total mass $M = m_1 + m_2 + m_{\text{eff}}$.
4. Hold the trolley on the tail end of the track. Click "Record" to start capturing data and release the trolley. Then you can acquire the velocity-time diagram of the trolley. The slope of the diagram is the acceleration a . Repeat steps above for numerous times. Then, record the experimental value of acceleration as $a = \bar{a} \pm \sigma_a$.
5. Change the weights on the trolley or the hook in the principle of keeping the total mass M constant. Repeat above steps.
6. Plot a vs. F . Make comparisons with the expected results.

➤ Motion on the inclined plane :**Preparation**

1. Hook the track rod clamp to the track (post in the side groove of the track) and mount it to stand rod. Fix the end side of the track on the table, and raise the pulley side of the track at an angle θ . Tight the screws in the stands. Hold the track carefully and safely during the whole process of adjustment. Thoroughly check whether the screws in the stand of the inclined plane are tightened to prevent damage of equipment. Loosened screws may cause the inclined plane to slip.
2. Use trigonometric formulas to calculate the inclined angle of the track θ as shown in figure 4. h_1 is the height of coordinate 1, h_2 is the height of coordinate 2, and d is the distance between these two coordinates.

$$\theta = \sin^{-1} \left(\frac{h_2 - h_1}{d} \right)$$

(Note : when using EXCEL for trigonometric formulas, the angle is in unit of radians)

3. Take an appropriate length of string. Tie one end of the string on O-ring and put the O-ring on the trolley's post. Tie the other end on the hook
4. Place the string on the Smart Pulley's groove and adjust the position of the O-ring. Make the string paralleled to the track.
5. Put yellow cushion pad on the floor right underneath of the hook and weights.

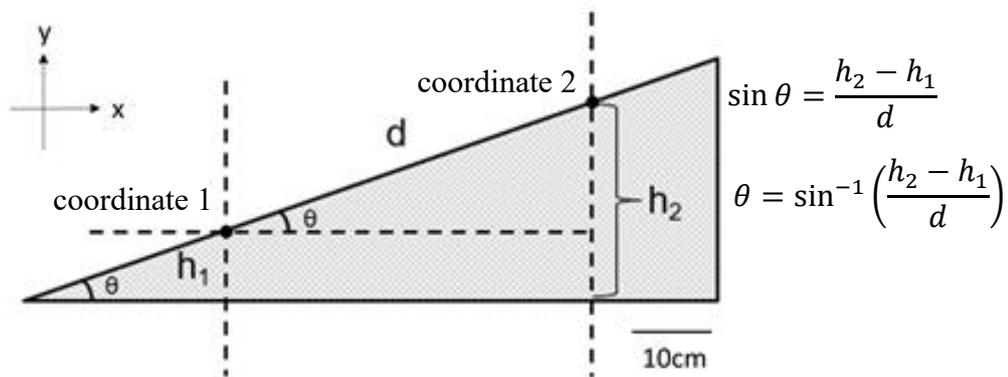


Figure 4. Schematics of a right triangle with an inclined angle θ .

C. Keep the system's force F and total mass M constant, and change the inclined angle θ

1. Weigh the hook and the weights on the hook using electronic scale and record the value as m_2 .
2. Weigh the trolley with loaded weights using electronic scale and record the value as m_1 .
3. Calculate the total mass $M = m_1 + m_2 + m_{\text{eff}}$.
4. Use trigonometric formulas to calculate the inclined angle of the track θ .
5. Hold the trolley on the tail end of the track. Click "Record" to start capturing data and release the trolley. Then you can acquire the velocity-time diagram of the trolley. The slope of the diagram is the acceleration a . Repeat steps above for numerous times. Then, record the experimental value of acceleration as $a = \bar{a} \pm \sigma_a$.
6. Change the inclined angle of the track θ . Repeat steps 4 and 5.
7. Make a table of the inclined angle θ and system's force F to rule out their relation.
8. Plot a vs. θ . Make comparisons with the expected results.
9. Plot a vs. $\sin \theta$. Make comparisons with the expected results.

Discussion hints :

1. In the horizontal motion, regarding the plots of $\frac{1}{a}$ vs. M and a vs. F , would you judge the existence of either static friction or kinetic friction between the wheels of trolley and the track and estimate its value.
2. What kind of friction is between string and pulley? Why the magnitude of friction is proportional to the acceleration of the trolley?
3. How to reduce the frictions in this experiment ?
4. If the experiment is done in the moon and the Mars, what will happen?

Circular motion

Objective :

Circular motion is one of basic motions in the world. For example, spin of electron in the atom and interaction between planets...etc. In this chapter, we would find out the relation of centripetal force with mass, radius, and rotation velocity by studying the circular motion in constant velocity.

Apparatus :

Rotational platform set, free mass, weights, level, Motor, DC power supply, electronic scales, wireless force/acceleration sensor (PS-3202), Computer (software Capstone).

Principle :

As shown in figure 1, a body of mass m is in constant horizontal circular motion due to a centripetal force.

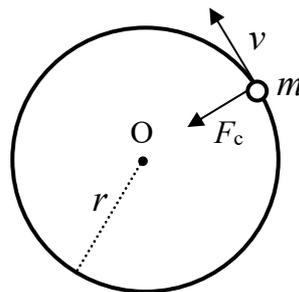


Figure 1. An overhead view of a particle m moving in a circle with the center O and radius r . v is the velocity of the particle and F_c the centripetal force that the particle experiences.

The centripetal force F_c can be expressed as:

$$F_c = m\omega^2 r = m \left(\frac{v}{r}\right)^2 r = m \frac{v^2}{r} \quad (1)$$

where v and ω are tangent speed and angular speed of the body, respectively.

In circular motion, the tangent speed v can be expressed as

$$v = r\omega = \frac{2\pi r}{T} \quad (2)$$

where T is period.

By equations (1) and (2), the object centripetal force F_c , the angular speed ω and the period T are expressed as follows

$$F_c = \frac{mv^2}{r} = \frac{m(r\omega)^2}{r} = mr\omega^2 = \frac{4\pi^2 mr}{T^2} \quad (3)$$

Figure 2 is the centripetal force experimental device diagram, m is the mass of rotation object (including the loaded weights). String connects the rotation object to wireless force/acceleration sensor (PS-3202).

The centripetal force is increased gradually with increasing the velocity of the body. The centripetal force can be expressed as

$$F_c = |\vec{F}_{\text{tension}} + \vec{F}_{\text{friction}}| = mr\omega^2$$

$$F_{\text{friction}} = \mu mg$$

where \vec{F}_{tension} is the tension of string measured by the force sensor, $\vec{F}_{\text{friction}}$ is the static friction between rotation body and track, μ is static friction coefficient between rotation body and track, g is the gravitational acceleration.

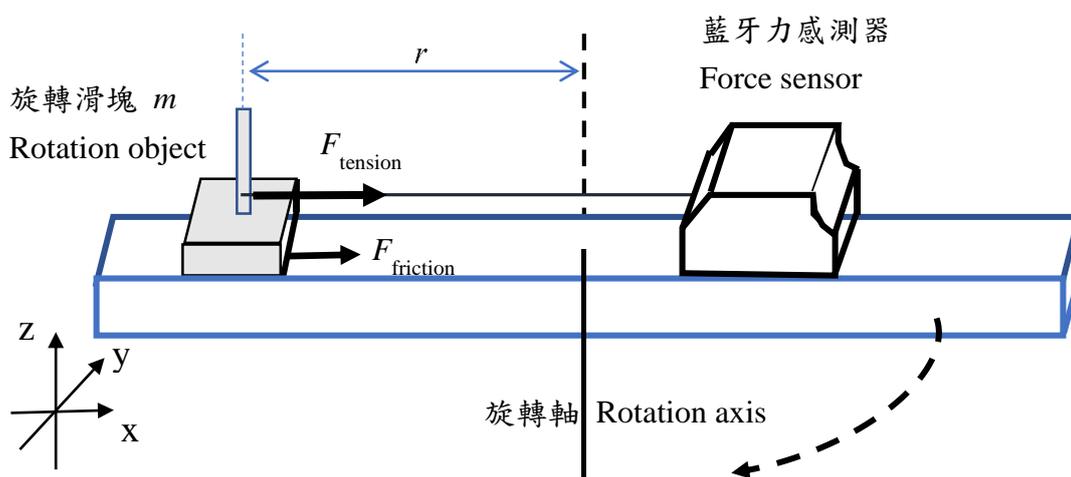


Figure 2. Centripetal force experimental device diagram. Dashed line is part of circular trajectory of the rotation object. m is the mass of rotation object (including the loaded weights). r is the radius of the rotation. F_{tension} is the magnitude of string tension and F_{friction} is the magnitude of static friction between rotation body and track.

Remarks :

1. Before rotating the track, make sure the rotation object and wireless force/acceleration sensor sit properly in the track. Also check all screws (including both ends of the track) are screwed tightly in preventing that the weigh will throw off the track and the unexpected accident occurs.
2. Do not pull or push the force sensor hardly to damage the apparatus.
3. The force value measured by force sensor is positive for pushing from cable to force sensor and negative for pulling from cable to force sensor, respectively.

4. Motor is driven by the DC power supply while the positive input (red) and negative input (black) of motor are connected to the positive output terminal (red) and negative output terminal (black) of the power supply. The applied voltage would not over 10.0 V.
5. Set output of DC power supply in a constant voltage mode (C.V.) : Turn four knobs counter-clockwise down to zero before powering it on. Switch ON. Turn the current coarse knob clockwise to the maximum and then, smoothly turn the voltage coarse knob clockwise to the desired value. Reversing the processes while turning off. (Front panel instruction is shown in appendix II)

Procedure :

➤ Preparation

1. Put the level on the track, and adjust the base screw to keep the track horizontal.
2. Loosen the screw of motor, connect rotation disk (underneath of the track) and motor by O-ring. Make sure the O-ring is taut while tightening up the screw. As completing the experiment, loosen all.
3. Make calibration of the force sensor using software capstone (see operation manual).
4. Adjust the position of the force/acceleration sensor to change the rotational radius. Unscrew the horizontal white PE screw, and then the vertical black HDPE screw. Relocate the force/acceleration sensor. Tight the vertical black HDPE screw and then, the horizontal white PE screw.
5. Angular speed is changed by DC voltage supplied to the motor.

A. The centripetal force vs. angular speed

1. Measure the mass of rotating body including loaded weights m .
2. Open software capstone and set all parameters on the entry screen based on the operation manual.
3. Put the rotation object and wireless force/acceleration sensor in the track. The later should be fixed in position and both are connected by a string. Measure the rotational radius r . All screws (including both ends of the track) need to be screwed tightly
4. Power on DC power supply and provide a proper voltage to trigger motor and rotate the track. (Note: When the voltage is low, track rotate slower and string may not be taut with a bigger error. Be aware that the voltage should be turned up gradually and would not over **10.0 V.**)
5. Wait for steady rotation, click “Record” to start capturing data. Simultaneously observe” angular velocity – time” and” tension of string – time” chart for 20 seconds - 30 seconds. Then change the output voltage to vary the angular speed. Repeat the above step for numerous experimental runs of different output voltages. Click “Stop” to finish collecting data. Turn down the output voltage to zero to stop rotating. (Note: the difference of two tensions is more than $> 0.5 \text{ N}$)
6. At all steady conditions, record the average angular speed ω and the average magnitude of tension F_{tension} .

- Plot F_{tension} vs. ω^2 and make linear fit to data. What is the y-axis intercept? Compare the slope of the linear fit with the expected value.

B. The centripetal force vs. rotational radius

- Measure the mass of rotating body including loaded weights m .
- Set the rotational radius r . (Suggestion: initial value is 10.0 cm)
- Power on DC power supply and provide a proper voltage to trigger motor and rotate the track.
- Wait for steady rotation, click “Record” to start capturing data. Simultaneously observe” angular velocity – time” and” tension of string – time” chart for 20 seconds - 30 seconds. Click “Stop” to finish collecting data. Turn down the output voltage to zero to stop rotating.
- Record the average angular speed ω and the average magnitude of tension F_{tension} .
- Change the rotational radius r (the position of force sensor). Keep the angular speed ω the same (adjust the output voltage V slightly), repeat the steps 3-5 for numerous runs of different rotational radii r .
- Plot F_{tension} vs. r and make linear fit to data. What is the y-axis intercept? Compare the slope of the linear fit with the expected value.

C. The centripetal force vs. mass of rotation object

- Measure the mass of rotating body including loaded weights m and the rotational radius r .
- Power on DC power supply and provide a proper voltage to trigger motor and rotate the track.
- Wait for steady rotation, click “Record” to start capturing data. Simultaneously observe” angular velocity – time” and” tension of string – time” chart for 20 seconds - 30 seconds. Click “Stop” to finish collecting data. Turn down the output voltage to zero to stop rotating.
- Record the average angular speed ω and the average magnitude of tension F_{tension} .
- Change the mass of rotating body m (loaded weights). Keep other parameters, r and ω , fixed. repeat the steps 2-4 for numerous runs of different masses m .
- Plot F_{tension} vs. m and make linear fit to data. What is the y-axis intercept? Compare the slope of the linear fit with the expected value.

Discussion hints :

- Can the static friction coefficient μ be derived in this experiment? Please elaborate on it.
- What influence would be to the experiment if the track is not completely horizontal? Please elaborate on it.

Collision in one dimension

Objective :

Place two trolleys on the nearly frictionless aluminum trajectory and carry out elastic collisions and inelastic collisions to verify the law of momentum conservation.

Apparatus :

Aluminum trajectory, trolley, webcam, weight, level, isotropic magnets, rubber band in U-shaped frame, Velcro tape, tip and clay, electronic scales, and computer (with software Logger Pro 3.16).

Principle :

When two particles collide in a closed (no mass enters or leaves) and isolated (no net external force acting on particles) system. The total momentum of the two particles remains the same before and after collisions, and thus the momentum is conserved. The total kinetic energy is conserved only in elastic collisions. Otherwise, inelastic collision occurs that part of energy is lost by heat or sound and the total kinetic energy decreases after collision. The following will briefly explain the changes of momentum and kinetic energy of the two trolleys before and after one dimensional elastic collisions, inelastic collisions and perfectly inelastic collisions on a nearly frictionless trajectory.

1. Elastic collision : both trolleys get closer and leave without physical contact. Total kinetic energy is conserved.
2. Inelastic collision : both trolleys get closer, in contact, and leave. Total kinetic energy is not conserved

Perfectly inelastic collision : both trolleys get closer and then, stick together. The lost of total kinetic energy is a maximum.

In this experiment, m_1 and m_2 are the total mass of trolley 1 and trolley 2 (including the loaded weights), respectively. Set the initial velocities of two trolleys as v_1 and v_2 , and the velocities of two trolleys after collision as u_1 and u_2 .

A. Elastic collisions

Fig. 1 shows an elastic collision between two trolleys on a nearly frictionless trajectory. Under this condition, the two trolleys must obey the laws of momentum conservation and kinetic energy conservation before and after collision. The equations can be written as

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

By solving these simultaneous equations, we would get

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$u_2 = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

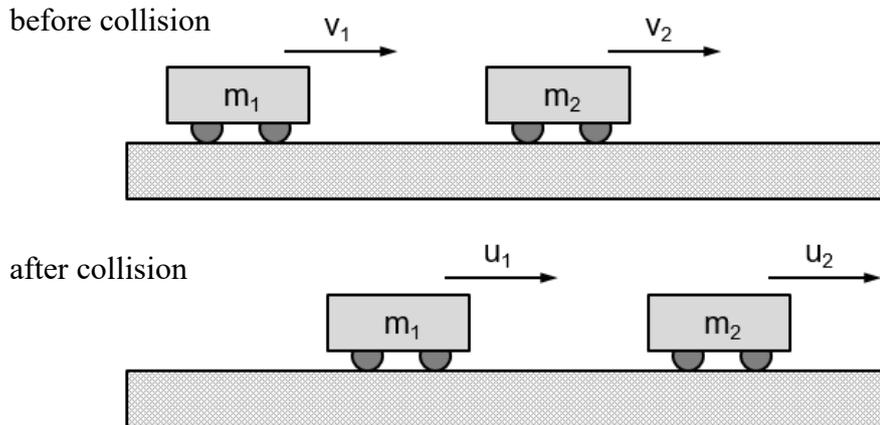


Figure 1. Schematics of an elastic collision between two trolleys. v_1 and v_2 are velocities of two trolleys before collision. u_1 and u_2 are velocities of two trolleys after collision.

If trolley 1 of mass m_1 in motion collides with trolley 2 of mass m_2 at rest ($v_2 = 0$), then

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$u_2 = \frac{2m_1}{m_1 + m_2} v_1$$

At the above condition that $v_1 > 0$ and $v_2 = 0$, there are three kinds of the elastic collisions.

Note : Here, we define all the velocities of the trolley: toward right hand side as positive, toward left hand side as negative.

1. As $m_1 = m_2$, $u_1 = 0$ and $u_2 > 0$. Two trolleys exchange their motions and velocities after collision.
2. As $m_1 > m_2$, $u_1 > 0$ and $u_2 > 0$. Two trolleys move in the same direction after collision.
3. As $m_1 < m_2$, $u_1 < 0$ and $u_2 > 0$. Two trolleys move in opposite directions after collision.

Inelastic collision

Fig. 2 shows a perfectly inelastic collision between two trolleys on a nearly frictionless trajectory. Perfectly inelastic collision happened when after collisions, two trolleys stick together and move in the same velocity ($u_1 = u_2 = u$) toward the same direction. Under this condition, only momentum was conserved between two trolleys before and after collision. The equations can be written as

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\Rightarrow m_1 v_1 + m_2 v_2 = (m_1 + m_2)u$$

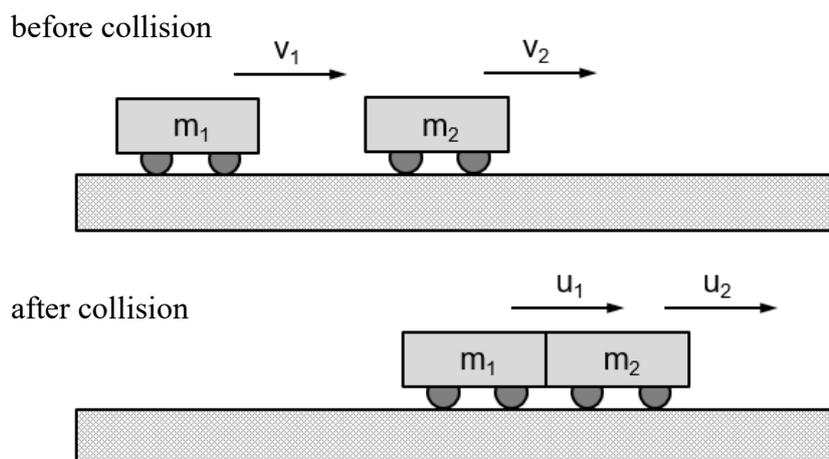


Figure 2. Schematics of the perfectly inelastic collision between two trolleys.

v_1 and v_2 are velocities of two trolleys before collision. u_1 and u_2 are velocities of two trolleys after collision.

If trolley 1 of mass m_1 in motion collides with trolley 2 of mass m_2 at rest ($v_2 = 0$), then

$$u = u_1 = u_2 = \frac{m_1}{m_1 + m_2} v_1$$

Recover Coefficient

The kind of collision can be judged by recover coefficient which is defined as the ratio of velocity difference of two trolleys after collision with respect to velocity difference of two trolleys before collision.

$$e = -\frac{u_2 - u_1}{v_2 - v_1}$$

1. When $e = 0$, it is a perfectly inelastic collision.
2. When $0 < e < 1$, it is an inelastic collision.
3. When $e = 1$, it is an elastic collision.

(a) The percentage of momentum lost

In elastic collisions and inelastic collisions, the total momentum of two trolleys should be conserved. During the collision, the trolleys may be affected by external force. That we can calculate the percentage of momentum loss according to the total momentum of the two trolleys before and after collision as following ΔP

$$\Delta P(\%) = \frac{(m_1 v_1 + m_2 v_2) - (m_1 u_1 + m_2 u_2)}{m_1 v_1 + m_2 v_2} \times 100\%$$

(b) The percentage of total kinetic energy lost

In inelastic collisions and perfectly inelastic collisions, the total kinetic energy of two trolleys would not be conserved. Now, we can calculate the percentage of total kinetic energy loss according to the total kinetic energies of the two trolleys before and after collision as following

$$\Delta E(\%) = \frac{(m_1 v_1^2 + m_2 v_2^2) - (m_1 u_1^2 + m_2 u_2^2)}{m_1 v_1^2 + m_2 v_2^2} \times 100\%$$

Remarks :

1. Level the trajectory, and tighten the screws before starting an experiment. Put cushion on both ends of the trajectory.
2. Connect the webcam to computer and place the webcam in front of the trajectory so that the webcam can film the motions of both trolleys on the trajectory. Make sure the view of webcam cover the whole trajectory and you can more data points.
3. Define all the velocities of the trolley: toward right hand side as positive, toward left hand side as negative.

Procedure :

➤ **Preparation**

1. Place the level on the trajectory. Adjust the screws on both sides underneath the trajectory to level it.
2. Place the trolley on the trajectory. Observe whether the trolley slides toward one side or the other of the trajectory. If so, please level the trajectory again until the trolley stops sliding. If it doesn't slide, proceed with the next correction.
3. Place the webcam in front of the trajectory (shown in Fig. 3). Use vision acquisition software to make sure you can acquire clear images of the two trolleys before and after collisions.

A. Elastic collision

Fig. 3 shows the top view of the set-up of trolleys' elastic collisions. m_1 and m_2 are the total mass of trolley 1 and trolley 2 (including the loaded weights and bumper), respectively. Webcam is the internet camera. Here, rubber bands and isotropic magnets can serve as bumpers. As two trolleys get closer and closer, they leave each other due to either the elastic interaction or the magnetic interaction.

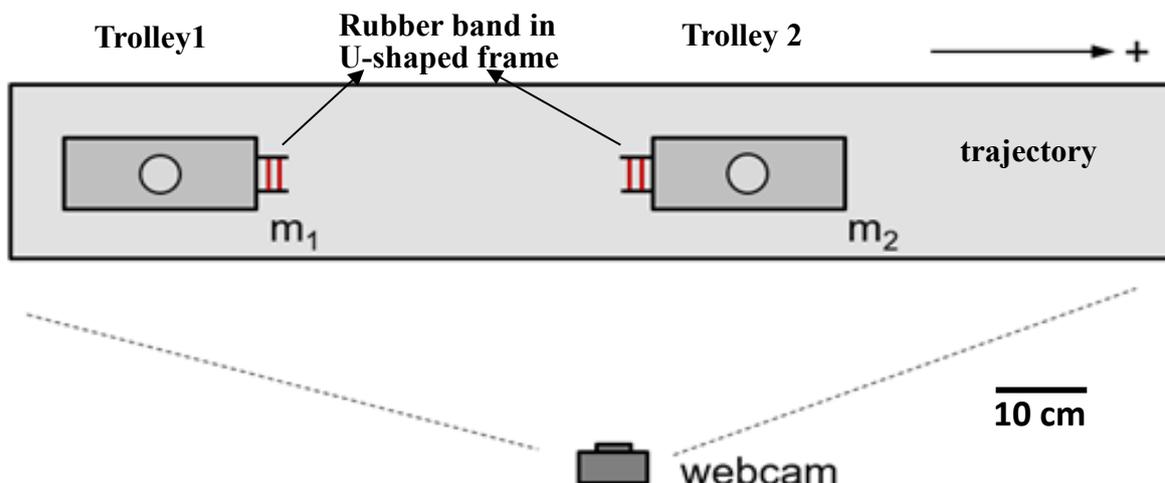


Figure 3. The top view of the set-up of trolley's elastic collisions. m_1 and m_2 are the total mass of trolley 1 and trolley 2 (including the loaded weights and bumper), respectively.

(a) Two trolleys with same mass

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the bumper) with an electronic scale. Keep the total masses of the two trolleys the same, $m_1 = m_2$.
2. Place the trolley 2 at rest ($v_2 = 0$) on the trajectory as shown in figure 3.
3. Use appropriate force to push trolley 1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps five times.
6. Calculate the total momenta and total kinetic energies of two trolleys before and after collision, and the recover coefficient as well as the percentage of total kinetic energy lost from data.

(b) A heavy trolley collides with a light trolley

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the bumper) with an electronic scale. Keep the total masses of the two trolleys satisfying $m_1 > m_2$.
2. Place the trolley 2 of mass m_2 at rest ($v_2 = 0$) on the trajectory as shown in figure 3.
3. Use appropriate force to push trolley 1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.

- Repeat above steps five times.
- Calculate the total momenta and total kinetic energies of two trolleys before and after collision, and the recover coefficient as well as the percentage of total kinetic energy lost from data.

(c) A light trolley collides with heavy trolley

- Weigh the total mass of the two trolleys (including the weights on the trolley and the bumper) with an electronic scale. Keep the total masses of the two trolleys satisfying $m_1 < m_2$.
- Place the trolley 2 of mass m_2 at rest ($v_2 = 0$) on the trajectory as shown in figure 3.
- Use appropriate force to push trolley 1. Keep capturing data until two trolleys collide.
- Acquire and record the velocities before and after collision with software.
- Repeat above steps five times.
- Calculate the total momenta and total kinetic energies of two trolleys before and after collision, and the recover coefficient as well as the percentage of total kinetic energy lost from data.

(d) Make comparisons among situations (a), (b), and (c). Describe how the masses of both trolleys influence their motions during the collision.

B. Inelastic collision

Fig. 3 shows the vertical view of the set-up of trolleys' elastic collisions. m_1 and m_2 are the total mass of trolley 1 and trolley 2 (including the loaded weights and bumper), respectively. Webcam is the internet camera. Here, Velcro tape and tip and clay can serve as bumpers.

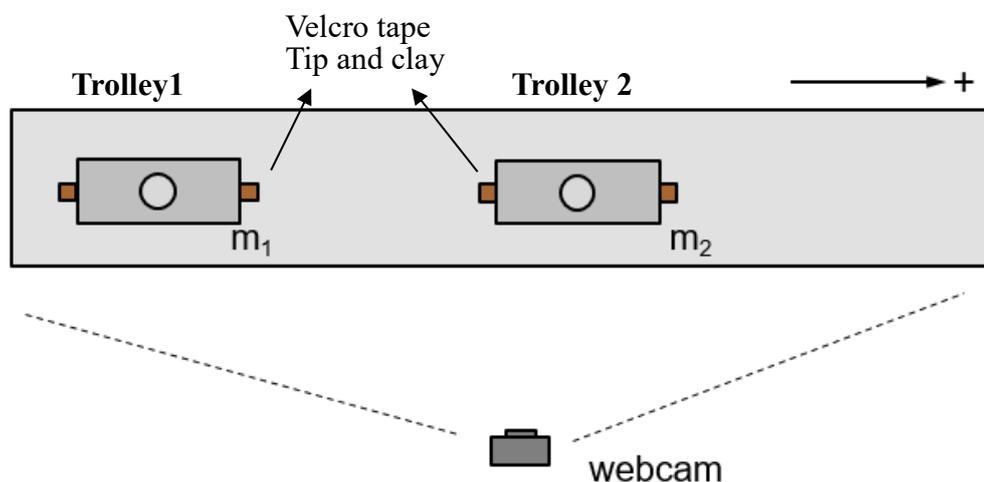


Figure 4. The top view of the set-up of trolley's inelastic collisions.

Keep the masses of both trolleys the same. The set of Bumpers includes rubber band in U-shaped frame, isotropic magnet (elastic collision), Velcro tape, tip and clay (perfectly inelastic collision), and magnet-Aluminum plate, Velcro- Aluminum plate, magnet-clay host (other inelastic collisions)

1. Weigh the total mass of the two trolleys (including the weights on the trolley and the bumper) with an electronic scale. Keep the total masses of the two trolleys the same, $m_1 = m_2$.
2. Place the trolley 2 at rest ($v_2 = 0$) on the trajectory as shown in figure 4.
3. Use appropriate force to push trolley 1. Keep capturing data until two trolleys collide.
4. Acquire and record the velocities before and after collision with software.
5. Repeat above steps five times for each set of bumpers.
6. Calculate the total momenta and total kinetic energies of two trolleys before and after collision, and the recover coefficient as well as the percentage of total kinetic energy lost from data.
7. Make comparison in the recover coefficient and the percentage of total kinetic energy lost among collisions of six kinds of bumper set.
8. In terms of motions of both trolleys during the collision, can the kinds of collision (elastic or inelastic) be attributed to the physical properties of bumpers?

Discussion hints :

1. Prove the velocity-mass equation of the two trolleys after collisions with laws of momentum conservation and kinetic energy conservation.
2. If the trajectory is not leveled during a collision, would total momentum of the system conserve?

Waves

Objectives :

Observe the propagation and resonance of the transverse wave on the chord wire. Moreover, investigate how the wave speed v is influenced by the tension τ and the line mass density of the string μ based on measurements of wave resonance.

Apparatus :

Function generator, mechanical driver, pulley (with support rods and hook), weight, strings, ruler, electronic scales.

Principle :

A. Traveling waves

A wave is a disturbance (change from equilibrium) of one or more physical quantities such that the values oscillate repeatedly about a stable equilibrium (resting) value. A wave pattern is formed in space at a snapshot. The wave that wave pattern moves at a velocity is called a traveling wave. For instance, a taut string is plucked and waves are generated on the string. When a pulse propagates to the right on a string, an observer on the ground sees the wave propagates but the medium on the string moves up and down repeatedly instead of propagating.

The wave speed depends only on the properties of the medium through which the wave travels. For a string of a uniform line density of mass under a fixed tension, the wave on the string travels at a constant velocity. Upon definition, in a time interval of one period T the wave travels through a displacement equal to one wavelength λ . Therefore, the wave speed, wavelength, and period are related by the expression

$$v = f\lambda$$

B. Standing waves

When waves are combined in the medium with boundary conditions, only certain waves of allowed frequencies can exist. The wave that wave pattern remains in space and its amplitude oscillates in time is called a standing wave. A standing wave has some locations, called nodes, that medium element never moves at all and amplitude is always zero. When an oscillating external force is applied at the resonant frequency of the medium, the medium will oscillate at a higher amplitude than when the same force is applied at other, non-resonant frequencies.

A standing wave on a string has an oscillation pattern with a stationary outline that results from the superposition of two identical waves traveling in opposite directions. Here, we create the standing waves on the string by mechanical driver with proper frequencies and measure corresponding wavelengths. From both quantities to calculate the wave speed.

C. The speed of wave

In figure 1 in a reference frame moving along with the pulse with the same speed, the tension τ makes the small segment Δs ($\Delta s = R \cdot 2\theta$), then the centripetal force can be further expressed as to do circular motion with radius R . Then the centripetal force will be expressed as

$$F_c = 2\tau \sin \theta \approx \tau(2\theta) = \tau \frac{\Delta s}{R} \quad (1)$$

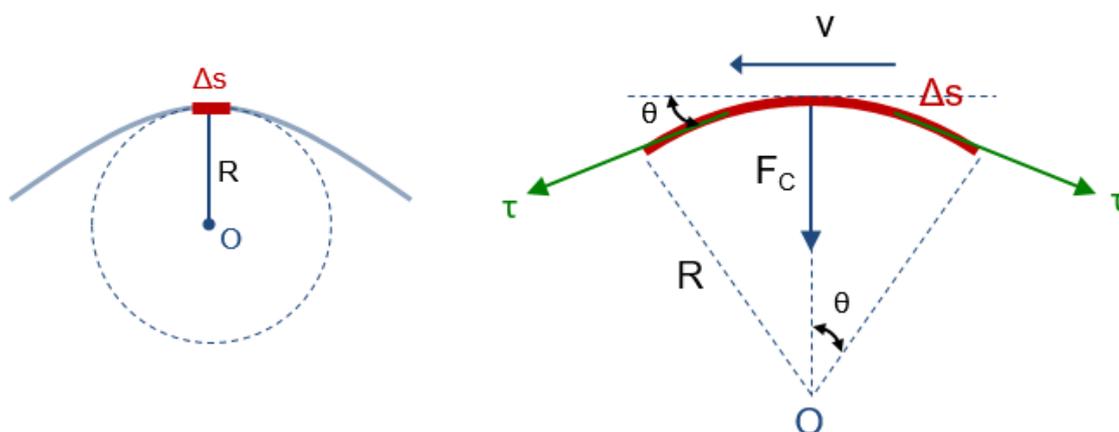


Figure 1 Illustration of force on small string segment. Length of string segment Δs , circular radius R , circular center O , open angle 2θ , the tension on the string segment τ , the centripetal force on the string segment F_c , and the wave speed v .

The mass of the string segment Δm can be expressed by its length Δs and the line mass density of the string μ

$$\Delta m = \mu \Delta s$$

At the instant the string element is located at the top of the pulse, it forms an approximate arc of a circle of radius R and moves as in a constant circular motion. The centripetal acceleration a can be written as

$$a = \frac{v^2}{R}$$

where v is the speed of the wave.

According to Newton's second law of motion, the centripetal force F_c can be written as

$$F_c = \Delta m a = (\mu \Delta s) \frac{v^2}{R} = 2\mu \theta v^2 \quad (2)$$

By equation (1) and (2), the centripetal force F_c can be expressed as

$$F_c = 2\tau \theta = 2\mu \theta v^2$$

and

$$\tau = \mu v^2$$

Therefore, the wave speed v can be expressed as

$$v = \sqrt{\frac{\tau}{\mu}}$$

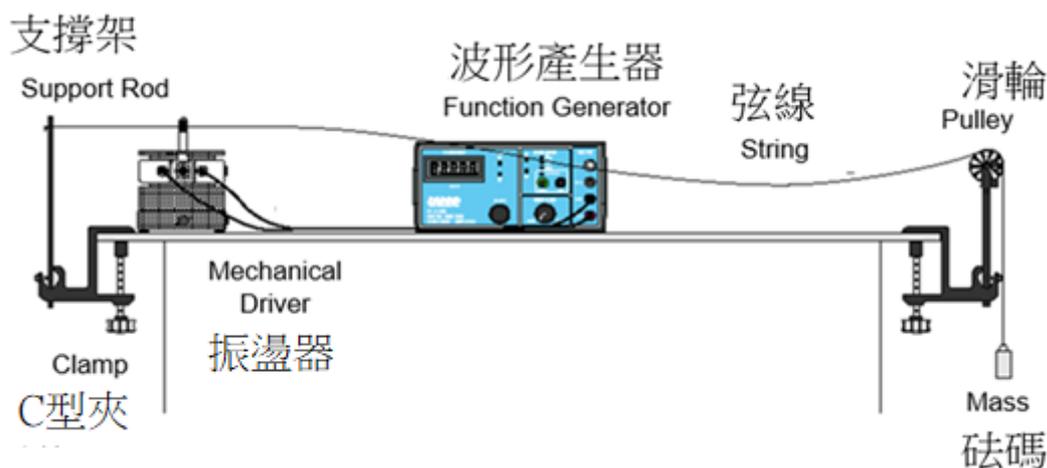


Figure 2. Experimental setup.

Remarks :

1. The mechanical driver should be close to the fixed end.
2. String should first pass through the hole of the drive arm of the mechanical driver and then tight the screw to fix string. One end of the string is attached to the support rod and the other end is tightly hooked.
3. “Unlock” the drive arm locking tab before turning on function generator. Shift the shaft of the mechanical driver back to “Lock” after you finish the experiment.
4. The weight placed on the direction of cut should be staggered hook.

Procedure :

Figure 2 shows the setup. Avoid putting a sideways force on the driver arm. If you are driving a wire or string that has tension, attach the end of the wire or string to a sturdy support as shown. The string pass over a pulley with the other end hang with weigh to provide the string tension. Adjust the position of pulley to keep string horizontal. Plug the output from your function generator into the banana plug receptacles on the front of the Mechanical Driver. Adjust the frequency and amplitude of the function generator to produce mechanical waves with the frequency and amplitude that you want.

A. Line mass density of the string μ

1. Measure and record the length of the string l .
2. Measure and record the mass of the string m_2 .
3. Calculate the line mass density of the string μ .

B. Harmonics of standing waves

1. Measure the mass of weights and hook m_1 .
2. Calculate the tension of the string $\tau = m_1 g$.
3. Set up the devices as fig.2 “Unlock” the drive arm locking tab and turn on function generator.
4. Adjust the oscillating frequency f and amplitude to obtain numerous standing waves. with systematical increasing f . Record the wavelength λ (number of nodes) and frequency f , respectively, for them.
5. Sketch the wave forms with indication of λ, f , and the wave speed by using $v = f\lambda$. Is the calculated value of wave speed consistent with the expected value $v = \sqrt{\frac{\tau}{\mu}}$?

C. Fix the density of the string μ and vary the tension of the string τ

1. Change the weight and repeat above steps to obtain the wave speeds for different m_1 .
2. Obtain numerous standing waves and record the wavelength λ and frequency f for them.
3. Calculate the wave speed by using $v = f\lambda$.
4. Plot v^2 vs. τ to show how the wave speed change with the string tension. Compare the experimental result with the expectation.

D. Fix the tension of the string τ and vary the density of the string μ

1. Change the string and repeat steps in part A to obtain the line mass density of the string.
2. Keep the tension of string $\tau = m_1 g$ fixed and repeat steps in part (B) to obtain the wave speed.
3. Repeat steps 2 and 3 to obtain the wave speeds for different μ .
4. Plot v^2 vs. $\frac{1}{\mu}$ to show how the wave speed change with the line mass density of the string. Repeat steps 2 and 3 to obtain the wave speeds for different μ

Discussion hints :

1. How does the amplitude change with increasing the numbers of nodes on the string? Please elaborate on it.
2. If the string vibrating direction is not horizontal, will the results of experiment be the same? Please elaborate on it.
3. Think about why we tie the string on the support rod and then penetrate through the mechanical driver, but it isn't tie the string on the mechanical drive directly? Please elaborate on it.

Thermodynamics

Ideal Gas Law Experiment & Absolute zero K

Part I: Ideal Gas Law Experiment

Objective :

Verify ideal gas law by varying the volume of Ideal gas law syringe and calculate the molar number n of air in the syringe with a fixed volume based on P , T , and V measurements.

Apparatus :

Gas law syringe (TD8596A), absolute pressure/temperature sensor (blue 、PS-2146), Air-Link(white 、PS-3200), Computer (software Capstone)

Principle :

A. Ideal gas equation

During 17th century, R. Boyle found the following relationship between pressure P and volume V at a constant temperature

$$PV = \text{const} \quad (\text{constant temperature}) \quad (1)$$

It is called Boyle's law. In an isothermal process, the gas pressure P is inversely proportional to the gas volume V .

Charlie (J. Charles) and Lussac (J. Gay-Lussac) found that at a fixed pressure P , the density of the gas volume V and the Celsius temperature t follow the relationship,

$$V = V_0 \left(1 + \frac{t}{273.15} \right) \quad (2)$$

where t is the Celsius temperature, and V_0 is the volume of gas at 0°C . That is, when the gas pressure P is constant, the increase in gas volume is $\frac{1}{273.15}$ times the amount of V_0 per Celsius.

From the gas temperature and the gas volume relationship can introduce a new temperature scale called the absolute temperature (absolute temperature):

$$T = 273.15 + t \quad (3)$$

From the above, 0°C corresponds to 273.15 K ; Celsius temperature $t = -273.15^\circ\text{C}$ equals the absolute temperature of 0 K, also known as absolute zero Kelvin.

Assume $T_0 = 273.15$ K, Eq.(2) can be rewritten as

$$\frac{V}{T} = \frac{V_0}{T_0} = \text{const} \quad (4)$$

When the gas pressure is constant, the gas volume of P and T is proportional to the absolute zero. That you combine (1) and (4) can find gas molecules must satisfy the following relationship:

$$\frac{PV}{T} = \text{const} \quad (5)$$

This equation is called the ideal gas state formula. Such so called ideal gases require that there is no gas molecular volume, no interaction among gas molecules, and the density is very low.

B. Molar number of gas molecules

From the ideal gas equation, the gas temperature under the standard condition is around $T_0 = 273.15$ K, the gas pressure of about $P_0 = 1.0$ atm, and the gas volume of about $V = 22.4$ L, which contribute to the equation (4) and get the gas constant R :

$$\begin{aligned} R &= \frac{P_0 V_0}{n T_0} = \frac{(1.0 \text{ atm})(22.4 \text{ L})}{(1.0 \text{ mol})(273.15 \text{ K})} \\ &= 0.082 \frac{\text{atm} \cdot \text{L}}{\text{mol} \cdot \text{K}} \\ &= 8.3 \frac{\text{N} \cdot \text{m}}{\text{mol} \cdot \text{K}} = 8.3 \frac{\text{J}}{\text{mol} \cdot \text{K}} \end{aligned}$$

Gas equation includes n moles gas can be written as

$$\begin{aligned} PV &= nRT \\ \Rightarrow V &= nR \frac{T}{P} \end{aligned}$$

From the ideal gas state equation, we can derive the gas volume V , temperature T and gas pressure P combine with following relation. Plot the figure between gas volume V (mL) and the gas temperature / gas pressure P (K/kPa), and its slope equals nR .

$$n = \frac{\text{slope}}{8.3} \times 10^{-3} (\text{mol})$$

Remarks :

1. Align the quick-release connector (white plastic) with the pressure port connector of the sensor. Push the connector onto the port, and then gently twist the connector clockwise to “lock” in place. (For unlocking, twist gently the connector counter-clockwise.)
2. To place plunger to the bottom of Gas Law Syringe, keep holding syringe by one hand (as shown on the right) and slap down on the plunger with the palm of your other hand.

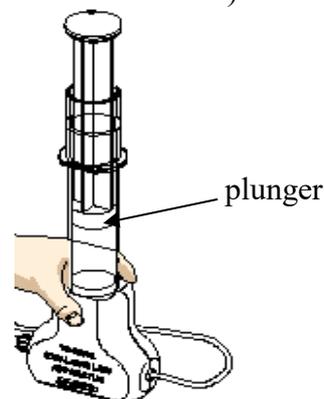


Figure 1. The sketch of Gas law syringe

Procedure :

A. Investigate the thermodynamic behaviors of ideal gas

1. Press the plunger down to the bottom of gas law syringe before connecting the gas law syringe to pressure sensor. Recorded at this time the air column volume V_2' in other words, the lower edge of the plunger (black rubber band) corresponding to the scale.
2. Pull up the plunger from syringe, and record the volume inside V_1' (around 35.0 mL~ 40.0 mL), in other words, the lower edge of the plunger (black rubber band) corresponding to the scale. Then wiring both sensors up.
3. Set all parameters and figure window in software “capstone” according to the operational manual.
4. Press start button to start acquiring data, and observe the 『 Pressure - Time 』 and 『 Temperature - Time 』 diagram for about 10 seconds. (figure 2, region I)
Fully compress the plunger quickly so that the stop is bottomed out. Hold this position until the temperature and pressure have equalized and are no longer changing.
Recorded at this time the air column volume V_2' .
[Note] The mark that was corresponded to the lower edge of the piston is V_2'
5. Observe 『 Pressure - Time 』 diagram, the air pressure will rise from P_1 to a certain maximum pressure P_{\max} , and then down to the steady pressure P_2 . (figure 2, region II)
6. Observe 『 Temperature - Time 』 diagram, the air temperature will rise rapidly from room temperature T_{room} to a maximum temperature T_{\max} , and then down to room temperature T_{room} . (figure 2, region II)
[Hint] Heat is escaped from gas law syringe.
7. When the 『 Pressure - Time 』 and 『 Temperature - Time 』 diagram do not change for about 10 seconds, please release plunger and allow it to expand back out on its own. (figure 2, region III)
8. After releasing plunger, observe the 『 Pressure - Time 』 and 『 Temperature - Time 』 diagram do not change for about 10 seconds, please stop recording data.

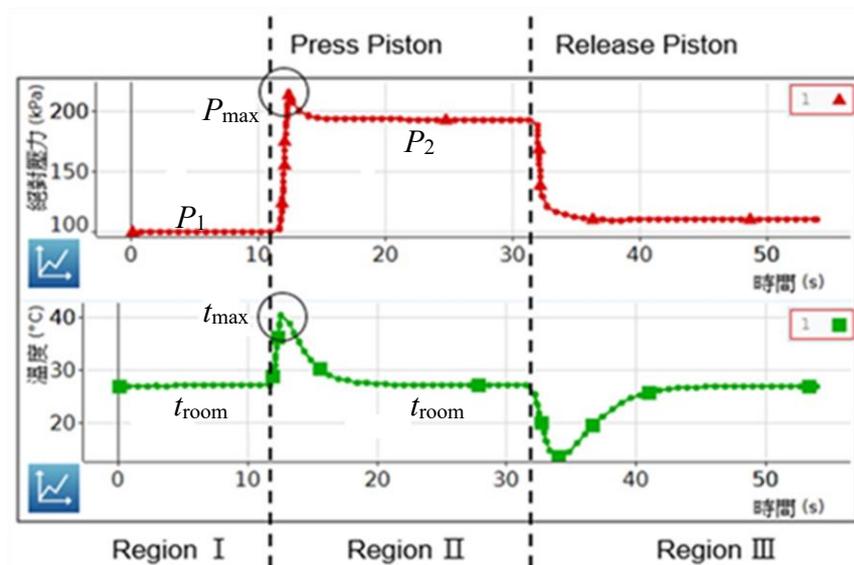


Figure 2. 『Pressure - Time』 and 『Temperature - Time』 diagram I.

Top: Pressure evolves with time. Bottom: Temperature evolves with time. Regions I, II, III are states corresponding the initial, pressing piston, and releasing piston, respectively.

(a) Measure volume of the transparent plastic tube V_0 in constant temperature

1. By figure 2, record air pressure P_1 and air temperature T_{room} in region I.
2. By figure 2, record air pressure P_2 in region II.
3. Based on Boyle's law ($P_1 V_1 = P_2 V_2$), calculate the additional volume V_r .

[Note] V_1 is the total volume of the gas law syringe before pressing the plunger.

$$(V_1 = V_1' + V_r)$$

V_2 is the total volume of the gas law syringe when the plunger is fully compressed. ($V_2 = V_2' + V_r$)

Additional volume such as connector and plastic tubing, V_r , is not included in the air column volume of Gas law syringe.

(b) Verify ideal gas law

1. By figure 2, record the maxima of air temperature T_{max} and air pressure P_{max} . Calculate and compare both values at the instants just before and after pressing piston, $\frac{P_1 V_1}{T_{\text{room}}}$ and $\frac{P_{\text{max}} V_2}{T_{\text{max}}}$.
2. Based on the measurements, (1) Pressure P (kPa) vs. t (s) and (2) Temperature T (°C) vs. t (s), describe how the gas state evolves with time. For instance, When the syringe volume is suddenly decreased, why both the temperature and the pressure go up to maxima and then, back down? When the plunger is released in the last part of the data run, what happens to the temperature? Why? Any constant pressure, constant volume, or adiabatic process is observed in this experimental run?

B. Obtain molar number n of gas molecules n

1. Before connecting the gas law syringe to pressure sensor, pull up the plunger and make gas column inside mark to 60.0 mL scale.
2. Click on software “Capstone” and key in all parameters.
3. Press start button to start acquiring data, and observe the 『Pressure - Time』 and 『Temperature - Time』 diagram for about 10 seconds
4. Hold the gas law syringe, and use the other hand to press the lower edge of plunger (black rubber band) to mark 55.0 mL, and **hold the position**.
5. Observe 『Pressure - Time』 diagram, the air pressure will rise from P_1 to a certain maximum pressure P_{\max} , and then down to the steady pressure.
6. Simultaneously observe 『Temperature - Time』 diagram, the air temperature will rise rapidly from room temperature T_{room} to a maximum temperature T_{\max} , and then down to room temperature T_{room} .
7. Wait again until the temperature and pressure have equalized and are no longer changing. Repeat steps 3~5. Record gas pressure value and gas temperature value of mark 60.0 mL, 55.0 mL, 50.0 mL, 45.0 mL, 40.0 mL, and 35.0 mL of 『Pressure - Time』 and 『Temperature - Time』 diagram.
8. When the 『Pressure - Time』 and 『Temperature - Time』 diagram such as figure 3 are obtained, stop recording data.
[Note] The volume of plastic tube V_r should be included. Try to calculate the value of V_r by $PV = nRT$ and compare it with the obtained value in part(A).
9. Plot $V(\text{mL})$ vs. T/P (K/kPa) and get the slope from Linear regression line. Calculate molar number of air n °

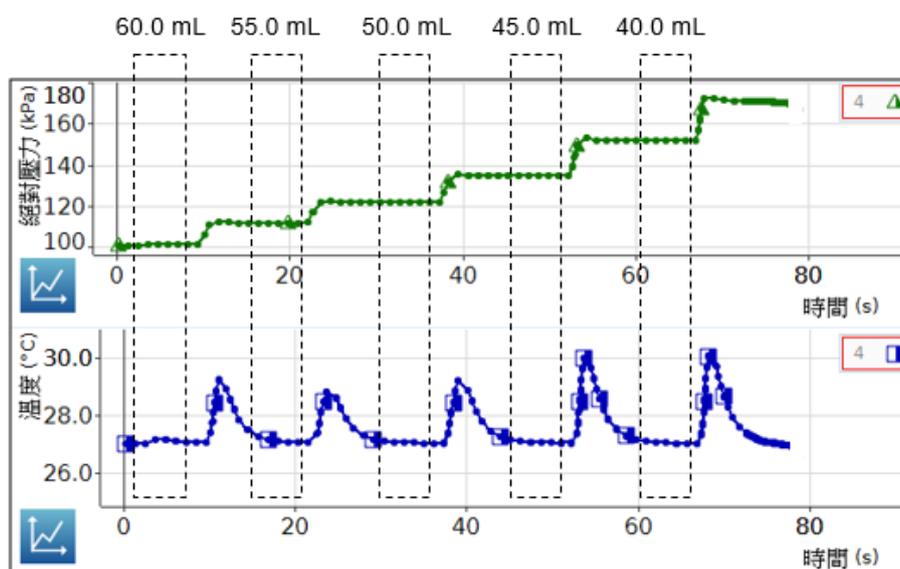


Figure 3 『Pressure - Time』 and 『Temperature - Time』 diagram II.

Part II : Absolute Zero K

Objective :

Use the Absolute Zero Apparatus to experimentally determine the temperature of 0 K.

Apparatus :

Absolute zero apparatus (TD8595), a digital thermometer, plastic beaker, stirring rod, electric kettle, absolute pressure/temperature sensor (blue · PS-2146), Air-Link (white · PS-3200), Computer (software Capstone)

Principle :

From the ideal gas equation, temperature T and gas pressure P can be represented as

$$T = \frac{V}{nR} P \quad (1)$$

where T is absolute temperature.

Furthermore, $T = 273.15 + t$ where t is Celsius degree. Eq.(1) can be rewritten as

$$273.15 + t = \frac{V}{nR} P$$

In the plot of t vs. P , y-axis intercept of the linear regression line is absolute zero K.

Remarks :

1. While using electric kettle, do not place the kettle near the corners and edges of table to prevent from accident. Be caution when the water is hot.
2. Align the quick-release connector (white plastic) with the pressure port connector of the sensor. Push the connector onto the port, and then gently twist the connector clockwise to “lock” in place. (For unlocking, twist gently the connector counter-clockwise.)

Procedure :

1. Make all connections and set up the experiment in the software “capstone”.
2. Fill proper volume of water into plastic beaker. (around 80°C - 90°C , 800 mL)
3. Stir hot water with a glass rod and measure the water temperature with a digital thermometer to check whether the thermal equilibrium is reached. When the water temperature is in thermal equilibrium, **sink the absolute zero ball completely in hot water.**
4. Wait until thermal equilibrium, press start button to start recording data. Observe the 『 Pressure - Time 』 and 『 Temperature - time 』 diagram about 10-20 seconds and then, stop recording. Remove the absolute zero ball from hot water.
5. Record the average absolute zero pressure P and temperature t of the ball at the stable condition. (temperature deviation is less than 0.2°C within 10 seconds, as shown in figure 4)
6. Decrease the temperature of hot water (pour cold water from another plastic beaker), repeat steps 3-5.
[Note] The water volume should not be more than 1000 mL.
7. Decrease the temperature of water farther by adding ice cubes. Repeat steps 3~5. At least eight experimental data points (different temperatures) are obtained.
8. Plot t ($^{\circ}\text{C}$) vs. P (kPa). Y-axis intercept of the linear regression line is experimental value of absolute zero K.

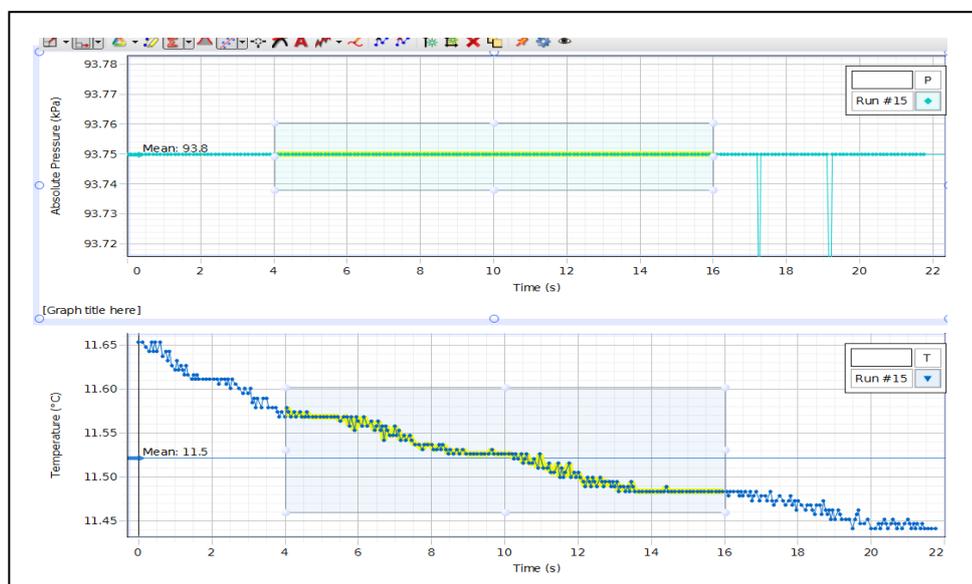


Figure 4. 『 Pressure - Time 』 and 『 Temperature - Time 』 diagram.

Discussion hints :

1. If the absolute ball is not completely submerged in the water, will it affect the accuracy of the experiment? Please elaborate on it.
2. Will the accuracy be influenced when the water temperature is lower than 30°C in the experiment? Please elaborate on it.

Appendix I

A. Physical properties of solid

matter	Mass density (g/cm ³)	Young's coefficient $\times 10^{11}$ (dyne/cm ²)	Line expansion coefficient α (10^{-6})	Specific heat (cal/g°C)	Resistance coefficient at 20°C ($\mu\Omega \cdot \text{cm}$)	Temperature coefficient of resistance ($10^{-3} [\text{K}^{-1}]$)
zinc	7.14	12.5	29.76	0.0925	5.8	4.2
aluminum	2.69	7.05	22.20	0.211	2.83	4.2
Glass (Crown)	2.4-2.5	6.5-7.8	8.97			
glass (Flint)	2.9-4.5	5.0-6.0	7.88			
gold	19.3	8.0	14.70	0.0309	2.44	4.0
silver	10.50	7.0	18.90	0.0560	1.59	3.1
rubber	0.91-0.96	0.048-0.052	65.7-68.6	0.4		
brass ¹	8.56	9.7-10.2	19.06	0.0925	5.0-7.0	1.42-2.0
bronze ²	8.7	8.08	18.44		13.0-18.0	
constantan ³	8.5	11.6	19.36	0.0946	17.0-41.0	0.04-0.38
copper	8.93	12.3-12.9	16.66	0.0919	1.72	4.3
tungsten	19.30		4.44	0.0321	5.5	5.3
manganin ⁴	8.15	12.4	18.1	0.097	44.0	-0.03-0.02
lead	11.34	1.62	27.09	0.0304	22.0	4.2
platinum	21.45	16.8	8.99	0.0316	10.6	3.8
Pig iron	7.8	10.0-13.0	10.61		57.0-114.0	
Wrought iron	7.8	19.0-20.0	8.5			
steel	7.8	19.5-50.6	11.4	0.11-0.13	10.0-20.0	5.0
nickel (97%)	8.9	20.2	12.79	0.1032	7.8	6.7

¹ 66%Cu, 34%Zn

² 85.7%Cu, 7.2%Zn, 7.1%Sn

³ 60%Cu, 15%Ni, 25%Zn

⁴ 84%Cu, 12%Mn, 4%Ni

B. Physical properties of liquid

matter	Mass density (g/cm ³)	Surface tension (dyne/cm)	Volume expansion coefficient at 20°C ×10 ⁻⁴	Specific heat (cal/g°C)	Heat of solution (cal/g)	Index of refraction
methanol	0.782	22.6	11.99	0.824		1.3290
ethanol	0.789	22.3	11.2	0.570	24	1.3625
ether	0.715	16.5	11.56	0.551		1.3538
Olive oil	0.918	32.0				1.4763
glycerol	1.260	63.4	48.95	0.576		1.4730
mercury	13.55 (20°C)	487.0	1819	0.033	28	
gasoline	0.878	26.0		0.511		~1.4
water	0.998 (20°C)	72.75	3.02 (20°C-40°C)	1.00	79.7	1.3332

C. Thermal conductivity κ (cal/(cm·°C·s))

matter	κ	matter	κ	matter	κ	matter	κ
aluminum	0.504	nickel	0.142	quartz (0-axis)	0.030	cotton	0.000 04
brass	0.260	platinum	0.166	quartz (1-axis)	0.016	soil	0.0013
copper	0.918	silver	0.974	silk	0.000 22	concrete	0.0022
gold	0.700	zinc	0.265	Wood flour	0.000 12	air	0.000 05
iron	0.144	tin	0.155	rubber	0.0006		
lead	0.083	glass	0.0024	crust	0.004		

D. Surface tension of water (in air) T (dyne/cm)

temperature (°C)	0	5	10	15	20	25	30	40	60	80	100
T	75.64	74.92	74.22	73.49	72.75	71.97	71.18	69.59	66.18	62.61	58.85

E. Viscosity of water η (dyne·s/cm²)

temperature (°C)	0	10	20	30	40	50
η	0.017 92	0.013 08	0.010 05	0.008 01	0.006 56	0.005 49

F. Viscosity of liquid η (dyne·s/cm², at 20°C)

matter	ethanol	ether	glycerin	gasoline
η	0.0172	0.002 33	14.560	0.1274

G. Mass density of air (kg/m³)

mmHg	690	700	710	720	730	740	750	760	770	780
0 °C	1.174	1.191	1.208	1.225	1.242	1.259	1.276	1.293	1.310	1.327
5 °C	1.153	1.169	1.186	1.203	1.220	1.236	1.253	1.270	1.286	1.303
10 °C	1.132	1.149	1.165	1.182	1.198	1.214	1.231	1.247	1.264	1.280
15 °C	1.113	1.129	1.145	1.161	1.177	1.193	1.209	1.226	1.242	1.258
20 °C	1.094	1.109	1.125	1.141	1.157	1.173	1.189	1.205	1.220	1.236
25 °C	1.075	1.091	1.106	1.122	1.138	1.153	1.169	1.184	1.200	1.215
30 °C	1.057	1.073	1.088	1.103	1.119	1.134	1.149	1.665	1.180	1.195

H. Index of refraction

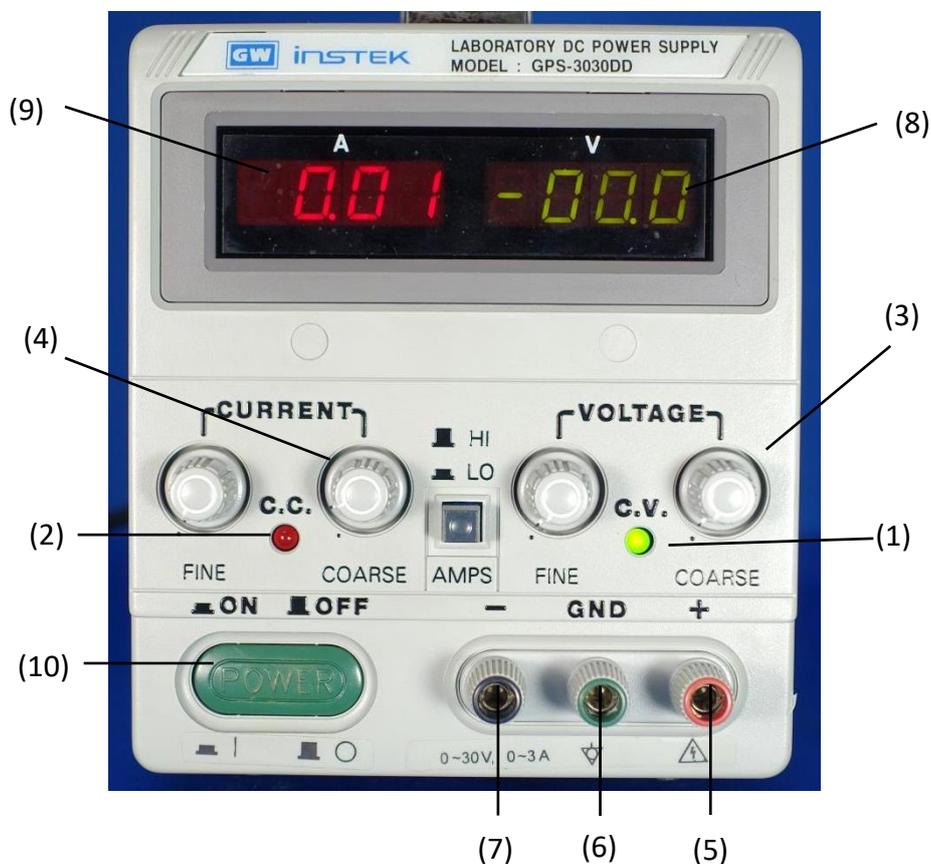
matter \ wavelength		C (H α) 656.3 nm	D (N α) 589.3 nm	E (H β) 486.1 nm
		water (18 °C)	1.3311	1.3332
ethanol (18 °C)		1.3609	1.3925	1.3665
Carbon disulfide (18 °C)		1.6199	1.6291	1.6541
glass (Crown)	light	1.5127	1.5153	1.5214
	heavy	1.6126	1.6152	1.6213
glass (Flint)	light	1.6038	1.6085	1.6200
	heavy	1.7434	1.7515	1.7723
calcite	normal light	1.6545	1.6585	1.6679
	unusual light	1.4816	1.4864	1.4908
crystal	normal light	1.5418	1.5442	1.5496
	unusual light	1.5509	1.5533	1.5589

I. Physical coefficients

Name	Symbol	Approximation	SIunit
Avogadro's number	N_A	6.022×10^{23}	mol^{-1}
Boltzmann's constant	$k = R / N_A$	1.381×10^{-23}	J/K
Coulomb-law constant	$k (=1/4 \pi \epsilon_0)$	9.00×10^9	$\text{N} \cdot \text{m}^2 / \text{C}^2$
Electron mass	m_e	9.109×10^{-31}	kg
Elementary charge	e	1.602×10^{-19}	C
Permeability constant	μ_0	$4\pi \times 10^{-7}$	H/m
Permittivity constant	$\epsilon_0 (=1/\mu_0 c^2)$	8.854×10^{-12}	F/m
Planck's constant	h	6.626×10^{-34}	J·s
Speed of light in vacuum	c	3.00×10^8	m/s

Appendix II

DC power supply front panel instruction :



1. Constant voltage indicator
2. Constant current indicator
3. Coarse control of voltage
4. Coarse control of current
5. Output terminal (“positive”)
6. Output terminal (“ground”)
7. Output terminal (“negative”)
8. Voltage meter
9. Current meter
10. Power switch